Spring 2012

Characterization of soil variability for reliability-based design

Sitenikechukwu Onyejekwe

Department: Civil, Architectural and Environmental Engineering

CHARACTERIZATION OF SOIL VARIABILITY FOR RELIABILITY-BASED DESIGN

by

SITENIKECHUKWU ONYEJEKWE

A DISSERTATION

Presented to the Faculty of the Graduate School of the

MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

In Partial Fulfillment of the Requirements for the Degree

DOCTOR OF PHILOSOPHY

In

CIVIL ENGINEERING

2012

Approved

Louis Ge, Advisor

Richard W. Stephenson

Ronaldo Luna

Bate Bate

David Rogers

ABSTRACT

A very carefully planned Missouri-wide field exploration and laboratory investigation program, with a focus on fine-grained soils, was executed with the aim of characterizing the variability of geotechnical parameters statistically with a view to increasing the use of reliability-based design (RBD) among geotechnical engineers. Geotechnical parameters were characterized in terms of both their first and second statistical moments and their coefficient of variation (COV). Their probability distributions and their scale of fluctuation, θ, were also determined. Correlations between difficult-to-obtain parameters and more easily-obtained parameters were developed and the degree of fit of study data to published empirical correlations was investigated.

Results of the analyses show that: COV and probability distribution of parameters are dependent upon the soil classification type and in-situ state; Field data, like CPTu data, which provide sufficient data to establish a well-defined parameter profile, are the best for determining θ; The Semivariogram Function (SVF) is better suited than the Autocorrelation Function (ACF) for the determination of θ from widely-spaced, non-continuous, irregular data obtained from laboratory tests; Considering the fewer number of data points from a dataset required (half that of SVF) for analysis with the ACF, the SVF is better for the determination of θ.

A framework which incorporates the spatial averaging effect of parameters based on the scale of fluctuation and variance reduction factor that are computed from widely-spaced irregular and non-continuous data was proposed. The application of this framework to RBD was also illustrated with examples.

ACKNOWLEDGMENTS

I am indebted to my advisors, Dr. Louis Ge and Dr. Richard Stephenson for their immeasurable guidance, support, and encouragement throughout the research. I also wish to thank members of my Advisory Committee, Drs. Ronaldo Luna, Bate Bate, and David Rogers, for their valuable advice, comments, and suggestions.

I wish to thank the Missouri University of Science and Technology (S&T) and the Department of Civil, Architectural, and Environmental Engineering (CArE) for the Assistantships I had during the course of my studies. I also wish to thank Missouri Department of Transportation (MoDOT) for providing funds for this research.

I acknowledge the support I got from the Principals and Research Assistants on the Geotechnical Research Team of the Missouri Transportation Institute (MTI)/ Missouri Department of Transportation (MoDOT) Transportation Geotechnics Research Program. The Principals include Drs. R. W. Stephenson, R. Luna, L. Ge, and N. Maerz from S&T, and Drs. J. J. Bowders, E. Loehr, B. Rosenblad, and W. Likos from University of Missouri-Columbia (MU) while the Research Assistants include Kyle Kershaw, Mulugeta Kebede, Kerry Magner, Xin Kang, Stephanie Rust, Omar Conte, and Daniel Weingart from S&T, and Dan Ding and Mark Pierce from MU.

I also acknowledge the support I got from fellow students (Shuying Wang, Dominica Cambio, Nick Rocco, Betty Hailemariam, Mingyan Deng, Abdalmajeed Ali) of the Geotechnical Engineering Program and the technical administrative staff of CArE.

Finally, I wish to thank my family and friends for their unflinching support throughout the research.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

Figure Page

LIST OF TABLES

1. INTRODUCTION

1.1. OVERVIEW

The technical superiority of reliability-based design (RBD) methods over the deterministic methods of design of geotechnical structures has been firmly established in literature (Smith, 1986; Harr, 1987; Haldar and Mahadevan, 2000; Baecher and Christian, 2003; Fenton and Griffiths, 2007). Notwithstanding this technical superiority, there has been a rather slow uptake of RBD in the geotechnical community. This has been attributed variously to the unfamiliarity of geotechnical engineers to statistics and the general perception that RBD requires more data (hence more testing, implying increased cost), more time (which translates to cost) and generally more effort than the deterministic design methods currently in use. Whatever the extra costs attributable to RBD, its advantages appears to outweigh those disadvantages. The advantages of RBD include: the higher level of certainty in the determination of design risk; higher level of certainty in terms of the safety of the overall design; and presentation of the safety of a design in terms (probability of failure instead of factor of safety) easily understood by the public.

Principal to the deployment of RBD is the determination of the statistics of geotechnical parameters and the probabilistic analysis of these parameters. The primary statistics required to take the maximum advantage of RBD are the mean, variance, and scale of fluctuation, θ. These statistics have been found to be not only site-specific but also dataset-specific. When data is available, the mean and variance are fairly easy to compute. Computing the scale of fluctuation is a bit complex, requiring more data and a well-defined soil profile. A number of these statistics are published in the literature. However, owing to the provenance of data (sourced from far and wide) from which they were computed, the range in the properties is quite wide.

A considerable amount of data, more than required for deterministic design, is required to establish a well-defined soil profile and compute the scale of fluctuation. Field tests like the cone penetration test (CPT) provide sufficient data to establish a well-defined profile but do not enjoy widespread use. The more prevalent standard penetration test (SPT), in which undisturbed samples for laboratory testing are not available, does not provide sufficient data to establish a well-defined profile required for the computation of the scale of fluctuation.

Another method of soil exploration, the continuous Shelby tube sampling method, has the potential of providing sufficient data to establish a well-defined profile required for the computation of the scale of fluctuation. With this method, tests can be assigned at a closer spacing (closer than SPT) and direct measurements of geotechnical parameters can be carried out. It should be noted that for soil types where Shelby tube sampling is neither feasible nor possible, field tests like the SPT can be specified at closer intervals to obtain sufficient data to establish a well-defined profile required for the computation of the scale of fluctuation.

1.2. OBJECTIVES

This study is primarily focused on the collation and analysis of geotechnical data to develop parameters and statistics required for RBD, and to demonstrate the application of these parameters and statistics in the analysis and design of geotechnical engineering structures.

Based on data obtained from laboratory tests on fine-grained soil samples obtained from the continuous Shelby tube sampling method and field CPTu investigation program in Missouri, the study reported herein is aimed at increasing the use of RBD among geotechnical engineers. The main objectives of this study are as follows:

- Characterize fine-grained soils in Missouri for reliability analyses;
- Develop an RBD framework incorporating the spatial variability of geotechnical parameters based on widely-spaced, non-continuous, irregular data obtained from laboratory tests on specimens obtained by the continuous sampling borehole exploration method; and
- Demonstrate the application of the proposed framework.

1.3. DISSERTATION ORGANIZATION

This dissertation is organized into seven sections. An introduction to the research topic is presented in Section 1. A comprehensive literature review is presented in Section 2. Section 3 presents a description of the methods used in acquiring data for the study. The methodologies employed and results of analyses in terms of second moment statistics, probability distribution, correlation of parameters, and spatial variability are given in Section 4. The results of the analyses are discussed in Section 5. Section 6 describes a framework that incorporates the spatial variability of geotechnical parameters in RBD and examples of its application. Finally, conclusions and recommendations are presented in Section 7.

2. LITERATURE REVIEW

2.1. INTRODUCTION

All natural soils are highly variable in their properties and rarely homogeneous. These properties vary inherently from point to point in the ground due to several reasons including the depositional environment, the degree of weathering, and the physical environment (Lumb, 1974; Elkateb et al., 2003a; Jones et al., 2002).

Variability is a major contributor to uncertainty in geotechnical engineering analyses. Uncertainty pervades many aspects of geotechnical engineering particularly in the characterization of soil properties. Uncertainty in geotechnical properties can be formally grouped into aleatory and epistemic uncertainty (Lacasse and Nadim, 1996; Whitman, 1996; DNV, 2007). Aleatory uncertainty represents the natural randomness of a property and, as such, is a function of the spatial variability of the soil property. Epistemic uncertainty results from lack of information and shortcomings in measurement and/or calculation; for example, the systematic error resulting from factors such as the methods of property measurement, quantity of available data and modeling errors. Human error would be considered a third source of uncertainty; however, it is not usually considered in uncertainty analyses because it is difficult to isolate and its effects on probability are usually included in compilations of statistics on aleatory uncertainty (Jones, et al., 2002). A schematic of the sources of uncertainty in geotechnical soil properties is shown in Figure 2.1.

The conventional tools for dealing with soil heterogeneity in the field of geotechnical engineering have been relying upon high factor of safety and local experience. This is an inconsistent measure of performance, which has led to a broad acceptance of a need to develop more reliable tools to incorporate soil heterogeneity in a rather quantitative scheme amenable to engineering design.

Since the performance of geotechnical structure depends on local extremes of the properties within a subsurface profile, it is important to characterize the soil profile probabilistically (Vanmarcke, 1977). The probabilistic characterization of soil profiles provides a format for quantifying geotechnical information regarding the subsurface conditions at a particular site, a basis for predicting the performance of a geotechnical engineering structure and for quantifying the probability of failure, and enables a geotechnical engineer to assess critically and compare various site investigation and testing programs and to evaluate their effectiveness (Jaksa, et al., 2000).

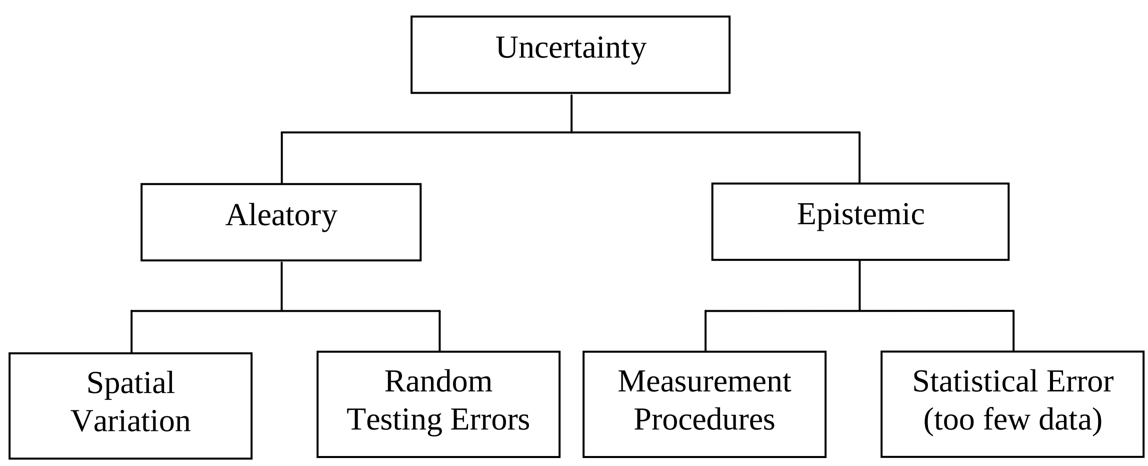


Figure 2.1: Sources of Uncertainty in Geotechnical Soil Properties (Adapted from Whitman, 1996)

2.2. SECOND MOMENT STATISTICS: MEAN, STANDARD DEVIATION, AND COEFFICIENT OF VARIATION (COV)

The most prevalent techniques for investigating uncertainty/material variability involve the combined use of probability and statistics (Lumb, 1974; Rethati, 1988; Phoon et al., 1995; Lacasse and Nadim, 1996; Phoon and Kulhawy, 1999a, 1999b; Duncan, 2000; Baecher and Christian, 2003; Ang and Tang, 2007; DNV, 2007; Uzielli et al., 2007; Fenton and Griffiths, 2008). In these techniques, the parameters are modeled as random variables. A random (or independent) variable is a quantity that is not known due to its random nature. The procedure generally involves defining the material properties by their statistics (principally their first and second moments): the mean, $\bar{x}$, and the variance, s^2, which define the probability density function and the coefficient of variation, COV. The mean of a data set is the sum of the data points in the data set divided by the total number of data points in the data set. The variance of a random variable is the mean value of the square of the deviation of that variable from its expected value or mean. The mean is the most common measure for the center of a data set. The variance is a measure of dispersion about the mean value of a data set. High values of dispersion mean higher uncertainty. Conversely, low values of dispersion mean low uncertainty.

Second moment statistics of geotechnical engineering parameters have been published by Kulhawy (1992), Cherubini and Giasi, (1993), Phoon et al., (1995), Lacasse and Nadim (1996), Phoon and Kulhawy (1999a), Duncan (2000), Jones, et al., (2002), Christian and Baecher (2003), Uzielli et al., (2007), among others. These published second moment statistics are useful for reference purposes. They are largely generic (due to widely sourced data) with a wide range of dispersion and hence may not represent the most economical or cost effective case. They should not be used uncritically for design for the following reasons (Uzielli et al., 2007): (a) The statistics of most geotechnical engineering parameters are dependent on in-situ state (which is not usually stated in the published statistics); (b) In most published statistics, the testing methods and/or procedures used in measuring parameters are not stated. It is possible to measure the same parameter using different methods and/or procedures resulting in different measured values as measurement occur in a different way; and (c) It is not possible to ascertain how homogeneous the soil is from which the statistics are calculated. Knowledge of this is required for these statistics to be applied correctly in other cases.

2.3. PROBABILITY DISTRIBUTION

Probability distributions, expectation, and moments are the basic statistical descriptors of a random variable. These descriptors can be used to estimate the variability of geotechnical soil probability density function (PDF) for a continuous random variable describing its probability distribution. There are a great number of distribution types used in mathematics and statistics. However, only a few are used in geotechnical engineering including normal, lognormal, exponential, gamma, uniform, and beta (Rethati, 1988; Christian and Baecher, 2003; Ang and Tan, 2007; Fenton and Griffiths, 2008).

Knowledge of the form of the probability distribution is not always necessary in the application of second-moment statistics to RBD; however, it may be necessary to plot the distribution to check the assumptions made about the distribution and check the reasonableness of the estimates (Jones et al., 2003; Stephenson 2009). The process of selecting and fitting a probability distribution that approximates a dataset best can be accomplished using many approaches and techniques. Two techniques commonly used are plotting a histogram of the data and choosing a distribution that appears to best-fit the data (histogram) and the Pearson's moment-based system (Harr, 1987; Rethati, 1988; Christian and Baecher, 2003).

The Pearson's moment-based system is an efficient system for the identification of suitable probability distribution based on third- and fourth-moment statistics, i.e. skewness, C_s and kurtosis, C_k, of the dataset. Skewness, C_s is a measure of the degree of asymmetry of a distribution around its mean. Positive skewness indicates a distribution with an asymmetric tail extending toward more positive values. Negative skewness indicates a distribution with an asymmetric tail extending toward more negative values. Kurtosis, C_k is a measure of the relative "peakedness" or flatness of a distribution compared with the normal distribution. Positive kurtosis indicates a relatively peaked distribution. Negative kurtosis indicates a relatively flat distribution. The Pearson's distribution space is presented in Figure 2.2 and the Pearson family types and the distributions included in them are presented in Table 2.1.

The goodness-of-fit of a chosen and fitted probability distribution to available data is tested by means of a number of approaches. These include visual inspections, the chi-squared (X^2) test, the Kolmogorov-Smirnov goodness-of-fit test and the normality test (Rethati, 1988; Christian and Baecher, 2003; Ang and Tan, 2007; Fenton and Griffiths, 2008).

Laboratory test results indicate that most soils can be considered as random variables having a normal or lognormal distribution (Lumb, 1966; Tan et al., 1993; Christian and Baecher, 2003; Elkateb et al., 2003a). Other distributions like the beta distribution have been used by other researchers (Ejezie and Harrop-Williams, 1984;

Rethati, 1988; Christian and Baecher, 2003; Elkateb et al., 2003a). However, best-fit probability distributions for geotechnical parameters are known to be primarily dataset-dependent, largely dependent on soil type and in-situ state. Owing to the fact that probability distributions are both parameter- and site-specific, it is impossible to select best-fit distributions for soil parameters in advance. For the forgoing reasons, published best-fit probability distributions, such as Corotis et al., (1975), Lacasse and Nadim (1996), and Nadim (2007) should not be accepted uncritically.

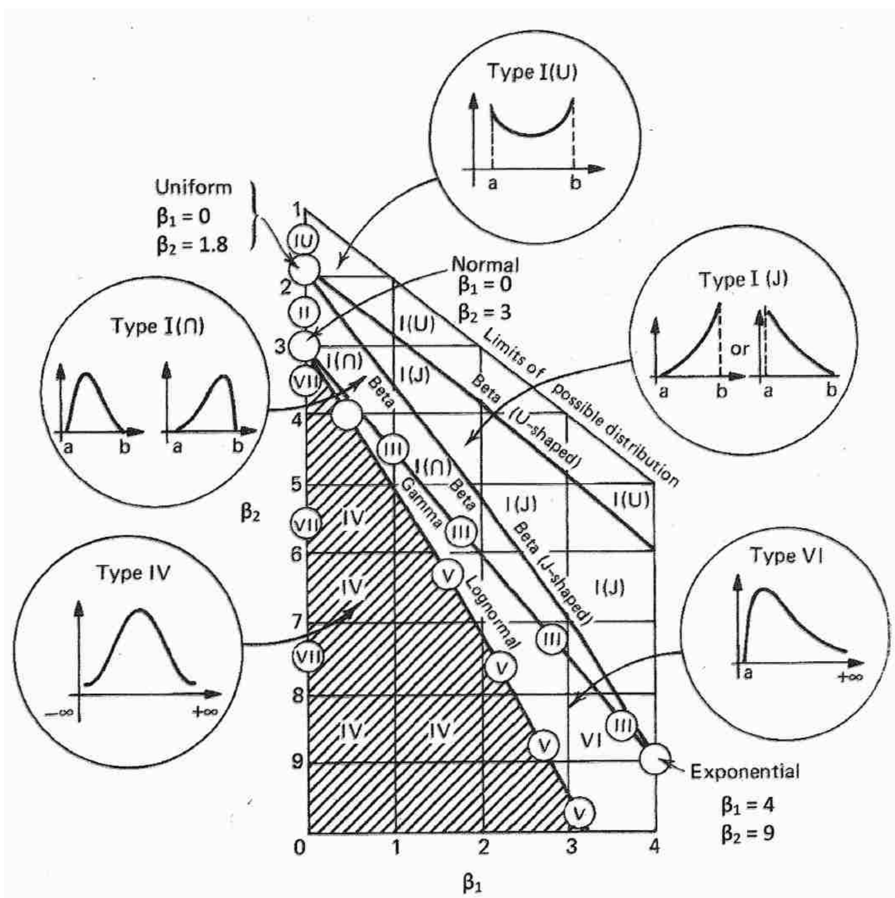


Figure 2.2: Space of Pearson's Probability Distributions (Adapted from Harr, 1987)

Table 2.1: The Pearson Family and the Equivalent Distributions

Pearson Family	Distribution
Type I	Beta
Type VI	Beta prime
Type IV	Cauchy
Type III	Chi-square
Limit of Type I	Continuous uniform
Type III	Exponential
Type III	Gamma
Type VI	*F*-distribution
Type V	Inverse-chi-square
Type V	Inverse-gamma
Limit of Type I, III, IV, V, or VI	Normal
Type VI	Lognormal
Type VII (= non-skewed subtype of type IV)	Student's *t*

The goodness-of-fit of a chosen and fitted probability distribution to available data is tested by means of a number of approaches. These include visual inspections, the chi-squared (X^2) test, the Kolmogorov-Smirnov goodness-of-fit test and the normality test (Rethati, 1988; Christian and Baecher, 2003; Ang and Tan, 2007; Fenton and Griffiths, 2008).

Laboratory test results indicate that most soils can be considered as random variables having a normal or lognormal distribution (Lumb, 1966; Tan et al., 1993; Christian and Baecher, 2003; Elkateb et al., 2003a). Other distributions like the beta distribution have been used by other researchers (Ejezie and Harrop-Williams, 1984; Rethati, 1988; Christian and Baecher, 2003; Elkateb et al., 2003a). However, best-fit probability distributions for geotechnical parameters are known to be primarily dataset-dependent, largely dependent on soil type and in-situ state. Owing to the fact that probability distributions are both parameter- and site-specific, it is impossible to select best-fit distributions for soil parameters in advance. For the forgoing reasons, published best-fit probability distributions, such as Corotis et al., (1975), Lacasse and Nadim (1996), and Nadim (2007) should not be accepted uncritically.

2.4. CORRELATION IN SOIL PROPERTIES

In geotechnical engineering practice, the use of correlations and empirical relationships provides a fast, cost-effective means of predicting the value of some parameter based on the value of some other (possibly more easily determined) parameters provided the appropriate correlations are employed. In probabilistic analysis, the quantification of the correlation between two or more soil properties provides a more realistic assessment of uncertainty in design parameters and an indication of the degree of independence between the parameters (Rethati, 1988; DeGroot, 1996; Uzielli et al., 2007).

The correlation between two or more soil properties has been found to be dependent in varying degrees on soil type, testing method used to obtain the numerical value of the parameter itself, and the homogeneity of the soil (Uzielli et al., 2007). A lot of correlations among soil properties have been published. A publication by Kulhawy and Mayne (1990) presents over 50 of such correlations. Considering the forgoing, published correlation models, most of which are site-specific, are most likely not appropriate for Missouri.

From the foregoing review of second-moment statistics (mean, standard deviation, coefficient of variation (COV); probability distribution) and correlation in soil properties, it can be concluded that second-moment statistics are largely dataset-dependent with data distribution largely dependent on soil type and in-situ state. Published values, with their wide range in values, most times do not sufficiently represent the local situation and, as such, may not result in efficient, cost-effective outcomes. Consequently, there is a need to develop site-specific second-moment statistics.

2.5. SPATIAL VARIABILITY ANALYSIS

The second moment-based techniques for the characterization of uncertainty in geotechnical parameters discussed above do not take into account the spatial variability of the parameters. Geotechnical parameters are known to show dependence both laterally and with depth. They vary spatially with a greater tendency to be similar in value at closely neighboring points than at widely spaced points. This is the reason second moment statistics alone are not enough to characterize uncertainty in geotechnical parameters (Lacasse and Nadim, 1996; Jones, et al., 2002; Uzielli, et al., 2007). Figure 2.3 illustrates why second moment statistics alone are not enough to characterize uncertainty in geotechnical parameters. In Figure 2.3, spatial data, simulated by Monte Carlo Simulation, is shown to have similar distributions (top and bottom left) but different magnitudes of spatial correlation: weak correlation (top right) and strong correlation (bottom right).

It is possible to model the spatial variability of a soil deposit in all directions in detail. However, this will require an extraordinarily high number of measurements so that it is impossible to achieve in practice. To bridge this gap in knowledge, the hypothesis of the randomness in the variation of soil properties is usually adopted (Baecher, 1982).

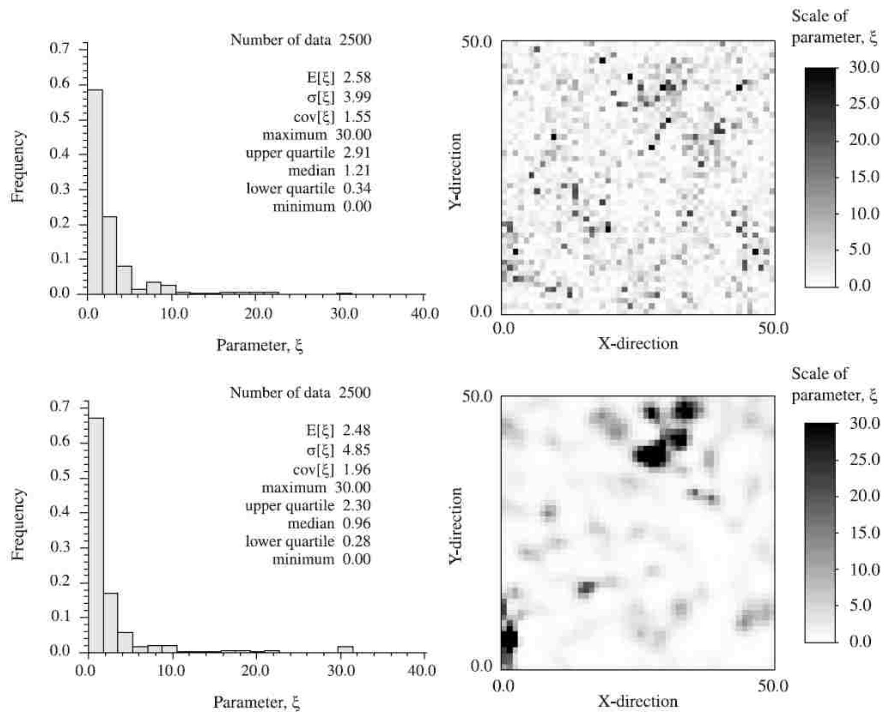


Figure 2.3: Spatial Data with Similar Distributions (Top and Bottom Left) but Different Magnitudes of Spatial Correlation (Adapted from El-Ramly et al., 2002)

Soil properties do not vary randomly in space. They vary gradually and follow a pattern that can be quantified using spatial correlation structures, where soil properties are treated as random variables (Elkateb, et al., 2003). The knowledge of spatial behavior of soil properties is often paramount in geotechnical analysis and design for the following reasons (Cherubini and Vessia, 2007; Uzielli, et al., 2007): (a) geotechnical design is based on site characterization, whose objective is to describe the spatial variation of compositional and mechanical parameters of soil; (b) the values of the parameters themselves very often depend on the in-situ factors (e.g. stress level, OCR, etc) which are related to spatial location; and (c) for large scale engineering endeavors such as dams and roads, it is generally expected that heterogeneous site characteristics will be revealed by investigations at spatially distant locations.

Geotechnical performance is often governed by spatial average soil properties, such as average shear strength along a pile shaft, or the average compressibility of a volume of soil beneath a footing in settlement calculation. The variability of soil properties averaged over a domain is less than that of their inherent variability or point properties (Vanmarcke, 1977; Phoon and Kulhawy, 1999a and 1999b). This is known as the averaging effect in spatial variability. The spatial averaging effect results in a reduction in the variance used in reliability-based geotechnical design (RBD). Neglecting this spatial averaging effect will lead to an overestimate of variability and lead to a very conservative design. Hence, knowledge of the spatial average of a soil parameter over an appropriate domain will be of primary interest.

The spatial variability of a parameter can be described statistically by the mean, variance, and the scale of fluctuation, θ (Vanmarcke, 1977). The scale of fluctuation defines the distances over which there is a significant correlation of material property values (Vanmarcke, 1977). Analysis of spatial variability of geotechnical parameters has normally involved the use of (a) random field theory and time series analysis (Lumb, 1974; Vanmarcke, 1977); or (b) geostatistics, as proposed by Matheron (1963). In both approaches, the spatial variability of the parameter is estimated from samples obtained from a population.

In both approaches, there is the requirement of the stationarity of data. Data are stationary if satisfying the following: (a) The mean is constant with distance (that is, no trend exists in the data); (b) The variance is constant with distance (homoscedastic); (c)

There are no seasonal variations; and (d) There are no irregular fluctuations. The requirement of the stationarity is more stringent for time series analysis than in geostatistics. In both random field theory and geostatistics, it is common practice to transform a nonstationary dataset into a stationary one by removing a low-order polynomial trend, usually no higher than a quadratic, using the method of Ordinary Least Squares, OLS (Jaksa et al., 2002; Uzielli et al., 2007). Stationarity is generally assessed by inspection of a scatter-plot of the data, the sample autocorrelation function, the experimental semivariogram, and/or the Kendal's τ test (Jaksa et al., 2000).

In random field theory and time series analysis, the principal tool for modeling spatial variability of geotechnical parameters and estimating of the scale of fluctuation, θ is the autocovariance, c_k or the autocorrelation, ρ_k , at lag, k. These are defined as:

$$c_k = \mathrm{Cov}(X_i, X_{i+k}) = E\left[\left(X_i - \mu_x\right)\left(X_{i+k} - \mu_x\right)\right] = E[X_i X_{i+k}] - \mu_x^2 \tag{1}$$

$$\rho_k = \frac{c_k}{c_0} \tag{2}$$

where X_i is the value of the property X at location, i; μ_x is the mean of property, X; E[…] is the expected value; c_0 is the autocovariance at lag 0; c_k is equal to c_{-k}; and ρ_k is equal to ρ_{-k}.

Since c_k and ρ_k can only be estimated from samples obtained from a population, the sample autocovariance at lag k, c_k^* and sample autocorrelation at lag k, ρ_k^*, are generally evaluated. The sample autocovariance function is the plot of c_k^* for lags, k = 0, 1, 2, … k, while the sample autocorrelation function, ACF, is the graph of ρ_k^* for the lags, k = 0, 1, 2, … k; where k is the maximum number of lags available. Generally, k = N/4 (Box and Jenkins, 1970; Anderson, 1976; Jaksa et al., 2000; Kelkar and Perez, 2002); where N is the total number of data points. Also, the sampling interval is constant (Box and Jenkins, 1970; Anderson, 1976; Box et al., 1994; Kelkar and Perez, 2002; NIST, 2010).

The sample ACF is generally evaluated using:

$$\rho_k = \frac{\sum_{i=1}^{N-k}(X_i - \mu)(X_{i+k} - \mu)}{\sum_{i=1}^{N}(X_i - \mu)^2} \tag{3}$$

The ACF has been the predominant method used in the investigation of the spatial variability of geotechnical parameters. It has been used by numerous investigators (Asaoka and A-Grivas, 1982; Vanmarcke, 1983; Baecher, 1986; Keavney et al., 1989; DeGroot and Baecher, 1993; White, 1993; Wickremesinghe and Campanella, 1993; DeGroot, 1996; Phoon and Kulhawy, 1996, 1999a, 1999b; Jaksa et al., 1997, 2000; Fenton, 1999; Cafaro and Cherubini, 2002; Baecher and Christian, 2003; Cherubini et al., 2007; Srivastava and Babu, 2009). In almost all cases the ACF was used in connection with continuous (in terms of how data was obtained, not mathematically continuous) data at close, regular and fixed intervals, using mainly CPT data.

The ACF is used to determine the correlation distance, which is the distance over which a property exhibits a strong correlation, and the separation distance at which the covariance function decays to a value of (1/e), where e is the base of the natural logarithm. In time series analysis, the most commonly used technique for determining the correlation distance is known as the Bartlett's limit (l_B), which corresponds to two standard errors of the estimates. The Bartlett's limit (l_B) is defined as (Jaksa et al., 2000; Uzielli et al., 2007):

$$l_B = \pm\frac{1.96}{\sqrt{N}} \tag{4}$$

where N is the total number of data points.

The scale of fluctuation, θ is determined by fitting of a number of models to the sample ACF. The correlation distance corresponding to the Bartlett's limit (l_B) is equal to the scale of fluctuation, θ (Jaksa et al., 2000; Uzielli et al., 2007). Various models have been used in literature (Tang, 1984; DeGroot and Baecher, 1993; Lacasse and Nadim, 1996; Rackwitz, 2000; Phoon et al., 2003; Uzielli et al., 2005). Some of the models used include the single exponential, cosine exponential, second-order Markov and squared exponential. The relationship between correlation length and scale of fluctuation for each model is presented in Table 2.2.

Geostatistics is based on regionalized variables having properties that are partly random and partly spatial and that have continuity from point to point (Clark, 1979). One of the basic statistical measures of geostatistics is the semivariogram, which is used to quantify the degree of spatial dependence between samples along a specific orientation and to presents the degree of continuity of the property in question.

Even though a regionalized variable is spatially continuous, its values can only be determined from samples taken from a population. Thus, in practice, the experimental semivariogram, γ^*, is estimated from the available data using standard relationships. The experimental semivariogram γ^* is defined by (Clark, 1979):

$$\gamma^{*}(h) = \frac{1}{2n(h)} \sum_{i=1}^{n(h)} [g(x_i) - g(x_i + h)]^2 \tag{5}$$

where *g* stands for the value of the parameter, *x* denotes the position of one sample in the pair and *x* + *h* the position of the other, and n is the number of pairs.

Table 2.2: Relationship Between Correlation Length, R and Scale of Fluctuation, θ (Adapted from Uzielli et al., 2007, Jaksa et al., 1999)

Model	Mathematical Function	Scale of Fluctuation (θ)
Triangular	$\rho_{\Delta z} = 1 - (\lvert\Delta z\rvert/a)$ for $\lvert\Delta z\rvert \leq a$ $= 0$ for $\lvert\Delta z\rvert \geq a$	*a*
Single Exponential (SNX)	$\rho_{\Delta z} = \exp(-\lvert\Delta z\rvert/b)$	2b
Cosine Exponential (CSX)	$\rho_{\Delta z} = \exp(-\lvert\Delta z\rvert/c)\cos(\Delta z/c)$	c
Second-order Markov (SMK)	$\rho_{\Delta z} = \exp(-\lvert\Delta z\rvert/d)(1+(\lvert\Delta z\rvert/d))$	4d
Squared Exponential (SQX)	$\rho_{\Delta z} = \exp(-(\lvert\Delta z\rvert/e)^2)$	$\pi^{0.5}e$
Notes: *a*, b, c, d, e = correlation length associated with model; Δz = lag length		

For an efficient experimental variogram, Kelkar and Perez (2002) suggest a maximum lag of half the maximum possible distance, and a minimum of 10 pairs for each lag within a region of stationarity.

In cases where the data are not distributed at regular intervals; therefore, a sufficient number of pairs for a precise interval cannot be obtained the equation for the experimental semivariogram γ^{*} is modified to include a lag tolerance, Δh. Hence to estimate the variogram at a lag distance, h, all the pairs within $h \pm \Delta h$ are collected. The modified equation for the experimental semivariogram γ^{*} is presented following (Kelkar and Perez, 2002; Clark and Harper, 2000).

$$\gamma^{*}(h \pm \Delta h) = \frac{1}{2n(h \pm \Delta h)} \sum_{i=1}^{n(h)} [g(x_i) - g(x_i + h)]^2 \tag{6}$$

The variogram has been used extensively in the investigation of the spatial variability of geotechnical parameters. It has been used by numerous investigators (Kelkar and Perez, 2002; Clark, 1979; Soulie, 1984; Clark and Harper, 2000; Jaksa et al., 1993; Soulie et al., 1990; Chiasson et al., 1995; Baecher, 1984; Matheron, 1963; Elkateb et al., 2003a, 2003b; Meek, 2001; Nobre and Sykes, 1992; Jaksa et al., 1997, 2000; McBratney and Webster, 1986; Pinnaduwa et al., 2003; Phoon et al., 2004; Unlu et al., 1990). In all cases, the variogram was used in connection with continuous data at close, regular and fixed intervals, mainly CPT data.

In the determination of the scale of fluctuation, a continuous theoretical semivariogram model is fitted to the experimental semivariogram (Clark, 1979). There are many theoretical semivariogram models available. These include the nugget effect, spherical, exponential, Gaussian, and hole effect models (Meek, 2001; Baecher and Christian, 2003; Kelkar and Perez, 2002; Deutsch, 2002; Isaaks and Srivastava, 1989). However, the most common of these is the spherical models (Kelkar and Perez, 2002; Meek, 2001). The relationship between range of influence and scale of fluctuation for each model is presented in Table 2.3. The experimental semivariogram is characterized by three parameters (Jaksa et al., 1997; Clark, 1979; Kelkar and Perez, 2002, Deutsch, 2002; Isaaks and Srivastava, 1989): (a) the nugget effect, *Co*, which is due to microvariabilities of the mineralization, and measurement errors; (b) the sill, *C* + *Co*, which measures, on the average, the squared difference between data pairs; and (c) the range of influence, *a*, which is the distance at which samples become independent of one another. A typical theoretical semivariogram is presented in Figure 2.4.

With the theoretical semivariogram model was fitted to the data, the parameters of the variogram are determined and used in computing the scale of fluctuation, θ. The scale of fluctuation, θ is defined by Cressie (1993):

$$\theta = 2\int_0^{\infty} \rho(h)\, dh \tag{7a}$$

where

$$\rho(h) = 1 - (\gamma(h) - c_0)/c \text{ for an experimental semivariogram} \tag{7b}$$

$$\hat{\rho}(h) = 1 - (g(h) - c_0)/c \text{ for a semivariogram model} \tag{7c}$$

Table 2.3: Relationship between Range of Influence and Scale of Fluctuation

Model	Mathematical Function	Scale of Fluctuation (θ)
Gaussian	$\gamma_h = C\,(1 - e^{-h^2/a^2}) + C_o$	$\pi^{0.5}a$
Exponential	$\gamma_h = C\,(1 - e^{-h/a}) + C_o$	2a
Spherical	$\gamma_h = C\left(\frac{3h}{2a} - \frac{h^3}{2a^3}\right) + C_o$; h ≤ a $\gamma_h = C + C_o$; h ≥ a	3a/4
Circular	$\gamma_h = C\,(1 - (2/\pi)(\cos^{-1}(h/a) - (h/a)(1 - (h/a)2)^{1/2})) + C_o$; 0 < h ≤ a $\gamma_h = C + C_o$; h > a	8a/π
Notes: *a* = range of influence; h = lag length		

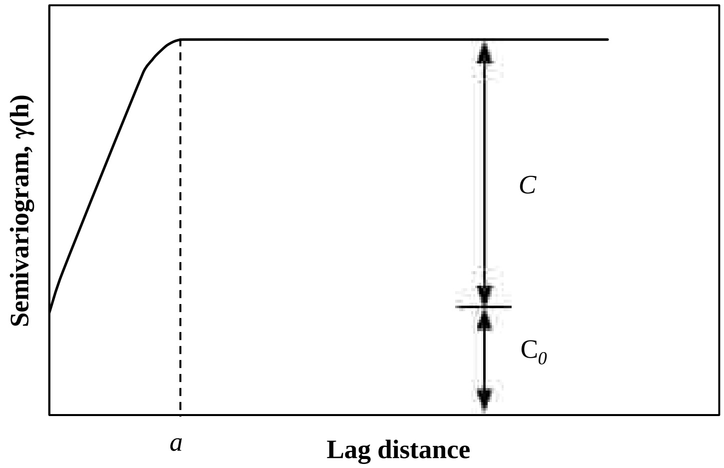


Figure 2.4: Typical Experimental and Theoretical Variogram

The determination of the scale of fluctuation, θ, is important in RBD. In time series (autocorrelation), and geostatistics analysis (semivariogram), the scale of fluctuation, θ is also evaluated by means of various models that relate the correlation distance and the range of influence, respectively to the scale of fluctuation, θ. For time series analysis, some of the autocorrelation models used include the single exponential, cosine exponential, second-order Markov and squared exponential models (Vanmarcke, 1983; Baecher and Christian, 2003, Uzelli et al., 2005; Fenton and Griffiths, 2008; Keaveny et al., 1989; Jones, et al., 2002). While for geostatistics analysis, some of the semivariogram models used include the nugget, linear, spherical, exponential, Gaussian, power, and hole effect models (Meek, 2001; McBratney and Webster, 1986; Elkateb et al., 2003a; Jones, et al., 2002; Kelkar and Perez, 2002, Deutsch, 2002; Isaaks and Srivastava, 1989).

2.6. VOLUME – VARIANCE RELATIONSHIPS

Volume – Variance relationships are analytical expressions used to obtain the variance of spatial averages of field data over certain volumes of interest. These spatial averages usually have a narrower probability distribution function than those usually associated with field data (Vanmarcke, 1977) and consequently a smaller variance. The scale of fluctuation is used in the spatial averaging of geotechnical properties to reduce their point variance.

Vanmarcke (1977) defined a variance reduction factor, $\Gamma^2(L)$ as the ratio of the point variance, σ_i^2 and the variance of the spatially averaged property, σ_L^2.

$$\Gamma^2(L) = \frac{\sigma_L^2}{\sigma_i^2} \tag{8}$$

The variance reduction factor depends on the averaging volume, type of correlation structure, and the limit of spatial correlation between field data. Analytical expressions for the variance reduction factor for exponential and Gaussian correlation structures were introduced by Vanmarcke (1983) while Elkateb, et al., (2003a) introduced an expression for circular correlation structures.

The expressions for the variance reduction factor for the exponential, Gaussian, and circular correlation structures are presented in Equations 9, 10, and 11, respectively.

$$\Gamma_T^2 = 2\left({}^{R}\!/_{T}\right)^2 \left({}^{T}\!/_{R} - 1 + e^{-{}^{T}\!/_{R}}\right) \tag{9}$$

$$\Gamma_T^2 = 2\left({}^{R}\!/_{T}\right)^2 \left[\sqrt{\pi} \cdot \left({}^{T}\!/_{R}\right) \cdot \xi\left({}^{T}\!/_{R}\right) - 1 + e^{-{}^{T}\!/_{R}}\right] \tag{10}$$

$$\Gamma_T^2 = 1 - {}^{T}/_{2a} + {}^{T^3}/_{20a^3} \qquad (11)$$

where Γ_T^2 = one dimensional variance reduction factor; R = autocorrelation distance; T = size of the averaging volume; $\xi\left({}^{T}/_{R}\right)$ = error function which varies from 0 to 1 as T increases from 0 to infinity; and *a* = spatial range.

Vanmarcke (1983) also suggested an approximation of the variance reduction function defined solely in terms of scale of fluctuation, θ and the averaging distance (that is, the distance over which a geotechnical property is averaged). This approximation is presented as follows:

$$\Gamma^2(L) = \left[\frac{\theta}{L}\left(1 - \frac{\theta}{4L}\right)\right]^{\frac{1}{2}} \qquad \text{for } L/\theta > 1/2 \qquad (12a)$$

$$\Gamma^2(L) = 1 \qquad \text{for } L/\theta \le 1/2 \qquad (12b)$$

Once the variance of the geotechnical property is updated, accounting for its spatial variation as stated above, the reduced property variance from Equations 12a and 12b is used in the limit state function under consideration, alongside the associated mean value and the values of any other probabilistic and/or deterministic parameters.

3. DATA ACQUISITION

3.1. INTRODUCTION

Data for this study were obtained principally from the Missouri Department of Transportation (MoDOT)/Missouri Transportation Institute (MTI) Transportation Geotechnics Research Program (MoDOT/MTI-TGRP), hereinafter referred to as TGRP, jointly executed by MoDOT, the Geotechnical Engineering programs of the Department of Civil and Environmental Engineering at the University of Missouri-Columbia (MU) and the Department of Civil, Architectural and Environmental Engineering at the Missouri University of Science and Technology (S&T) and the Geological Enginnering program of the Department of Geological Sciences and Engineering at the Missouri University of Science and Technology (S&T). Other sources of data include MoDOT tests (on samples from sister boreholes) carried out as part of the TGRP and MoDOT's geotechnical database. Sister boreholes are boreholes drilled a few feet from the Research (MU and S&T) holes by MoDOT.

3.2. FIELD EXPLORATION

Pursuant to the execution of the TGRP, a very carefully planned field exploration and laboratory investigation program was executed. Field exploration for soil samples and cone penetrometer (with porewater pressure measurements) (CPTu) soundings were carried out in three out of the four (except Ozark Highlands) geological zones in the state of Missouri (see Figure 3.1) at the following locations (Saville and Davis, 1962):

- Warrensburg (Johnson County) – Western Plains;
- New Florence (Montgomery County) – Glaciated Plains;

- St. Charles (St. Charles County) – Glaciated Plains; and
- Hayti (Pemiscot County) – South Eastern Lowlands.

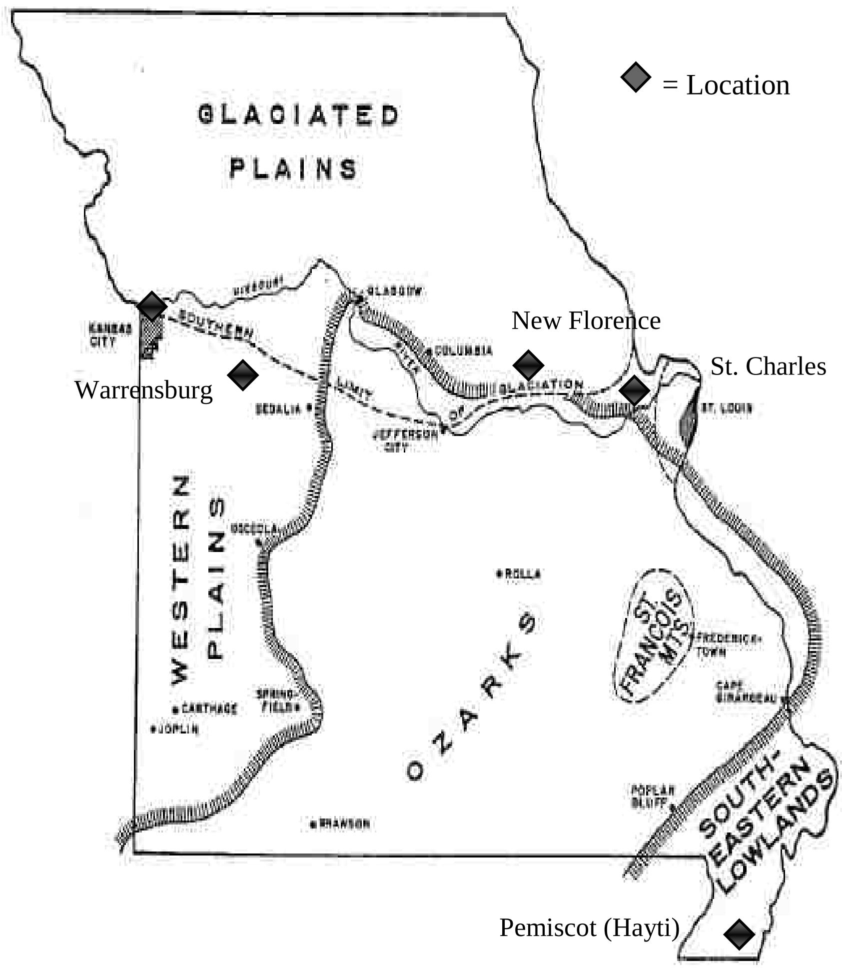


Figure 3.1: Major Geologic Regions of the State of Missouri (Adapted from Saville and Davis, 1962)

The soils in Warrensburg are located in the Central Lowland physiographic province. They are generally composed of sandy clayey residuum, one to three meters, derived from the in-place weathering of Pennsylvanian age shales and sandstone beds of the Weldon River-Warrensburg-Moberly channel sandstone (Whitfield, 1982; Middendorf, 2003; MODGLS, 2007). This channel fill runs northward, towards the Missouri River, and was incised in older shales and sandstone beds of the Cherokee Group, also of Pennsylvania age.

The St. Charles County site near Wentzville is located in the Central Lowland physiographic province. It is underlain by Holocene alluvium and glacial outwash deposits up to 35 m thick. This heterogeneous assemblage includes mixtures of overconsolidated clay, silt, sand, and gravel deposited by glacial outwash, lying upon the Mississippian age St. Louis Limestone, which is intensely karstified. Many of the surficial clays are highly plastic and are often removed by excavation before development to obviate their tendency to shrink and swell noticeably. These surficial "pockets" of clay are believed to be associated with the last glacial advance, which reached and dammed the mouth of the Missouri River.

The New Florence site in Montgomery County is located in the Central Lowland physiographic province. It is underlain by cherty clay, developed as residuum, one to 12 m thick, on the underlying Mississippian age Keokuk and Burlington Limestones, which containing abundant chert horizons. Drainages in the area floor in the older limestones, while the cultivated uplands are underlain by the younger, Pennsylvanian age shale and sandstone beds of the Cherokee Group. Soils derived from the shales in the Cherokee Group can be of moderate to high plasticity, and thereby, prone to seasonal shrink and swell behavior.

The Pemiscot County site lies within the Coastal Plain Province of the Upper Mississippi Embayment (Fenneman, 1938), near Caruthersville, MO. The area is underlain by point bar deposits (or meander scrolls) of the modern Mississippi River, classified by Saucier (1994) as Meander Belt 1, the youngest of six recognized belt systems along the modern Mississippi River flood plain. These Holocene age sediments are about 30 m thick, underlain by Pleistocene gravels and braid-bar deposits. The upper 10 m of this alluvium is typically underconsolidated, consisting mainly of overbank silts, which becomes increasingly sandy with depth.

Considering the different characteristics of the four sites, particularly that of the Pemiscot site which due to the high compressibility of the soils in the area is considered an outlier in the state, geotechnical parameters from each site was analyzed separately.

A special protocol, which largely followed best practice in geotechnical exploration and testing (Ladd and Lamb, 1963; Arman and McManis, 1976; Hight, 2001; Ladd and DeGroot, 2003; Stephenson, 2009a), was implemented to reduce disturbance to the samples/specimens at all stages: exploration and sampling; transportation and storage; extrusion and trimming; and testing. A summary of the field exploration conducted pertinent to this study is presented in Table 3.1. Table 3.1 presents on the bases of location, the number of exploratory boreholes, the depth range, the number of Shelby tubes obtained and the number of CPTu soundings and their depth range. Also presented in Table 3.1 is the total number of exploratory boreholes, the total number of Shelby tubes obtained, and the total the number of CPTu soundings statewide.

A description of the special protocol implemented to reduce sample disturbance to a minimum during field exploration, transportation, storage and testing as well as the testing protocols adopted for the TGRP are presented in Appendix A. Field exploration to obtain laboratory test specimens and field testing at each location was conducted within a two-week interval and hence are considered time invariant as suggested by Rethati (1988).

Table 3.1: Summary of the Field Exploration Conducted Pertinent to this Study

Location	Boreholes*			CPTu Soundings	
	Number	Depth Range (ft)	No. of Tubes	Number	Depth Range (ft)
Warrensburg	4	35 – 47.5	29	3	38.9 – 52.3
St. Charles	6	35 – 37.5	49	4	43.8 – 50.0
New Florence	9	7.5 – 35.1	65	5	16.9 – 36.1
Pemiscot	4	50 – 62.5	54	5	49.5 – 79.2
Total	23		197	17	
Notes: * = Includes Sister Holes, Boreholes drilled a few feet from the Research (MU and S&T) Holes					

3.3. FIELD TESTING

Field testing for this study was carried out by the Field Testing team of the TRGP led by Dr. Norbert Maerz.

The CPTu soundings were conducted using a Hogentogler all-terrain cone penetration rig using continuous, hollow-stemmed push rods to advance the seismic piezocone tip. The piezocone data collection equipment consists of a 60° cone, with 10 cm^2 base area, and a 150 cm^2 friction sleeve located above the cone (Maerz and Magner, 2010). The position of the porous filter for measurement of pore pressure (u_2) was behind the cone. Continuous data collection and sampling was performed by the Hogentogler

EFCS4 computer system as the cone tip was advanced at a rate of 20 mm/s ± 5 mm/s (rate is set by equipment) (Maerz and Magner, 2010).

Further analysis of the data from the Hogentogler EFCS4 computer system to derive geotechnical parameters was carried out with the CPTu data analysis and interpretation software, CPeT-IT version 1.6 (Geologismiki, 2011) (Maerz and Magner, 2010). The CPeT-IT software is the product of the collaboration between Geologismiki, Gregg Drilling, Inc, and the research group lead by Dr. Peter K. Robertson (Lunne, et al., 1997) (Geologismiki, 2011). CPeT-IT takes CPT data and performs basic interpretation in terms of normalized soil behavior type (SBTn) and various geotechnical soil and design parameters using current published correlations based on the comprehensive review by Lunne et al., (1997), as well as recent updates by Dr. Robertson (Geologismiki, 2011). Additional details on CPeT-IT software and its use in this study can be found elsewhere in Geologismiki (2011) and Maerz and Magner (2010), respectively. Some pertinent data and the correlations used in the determination of parameter values excerpted from the CPeT-IT software manual are presented in Appendix B.

3.4. LABORATORY TESTING

Laboratory testing was conducted according to the provisions of the appropriate and applicable American Society for Testing and Materials (ASTM) standards. The laboratory testing program included the following:

- Natural water content
- Atterberg limits
- Hydrometer analysis

- Consolidation (both Constant Rate of Strain, CRS and Incremental Loading, IL)
- Unconsolidated Undrained triaxial shear strength
- Consolidated Undrained triaxial shear strength

Natural moisture content determinations were according to the provisions of ASTM D2216 (2005): Standard Test Methods for Laboratory Determination of Water (Moisture) Content of Soil and Rock by Mass. Trimmings from the preparation of test specimens for the consolidation and strength tests were used in the natural moisture content determinations.

Atterberg limits determinations were generally according to the provisions of the following ASTM standards: (a) Liquid Limit: Section 11 (Multipoint Liquid Limit – Method A) of ASTM D4318 (2005): Standard Test Methods for Liquid Limit, Plastic Limit, and Plasticity Index of Soils; (b) Plastic Limit: Section 15 (Hand Method) of ASTM D4318 (2005): Standard Test Methods for Liquid Limit, Plastic Limit, and Plasticity index of Soils; (c) Plasticity Index: ASTM D4318 (2005): Standard Test Methods for Liquid Limit, Plastic Limit, and Plasticity Index of Soils; (d) Moisture Content: ASTM D2216 (2005): Standard Test Methods for Laboratory Determination of Water (Moisture) Content of Soil and Rock by Mass. However, in this study, to ensure moisture equilibrium within the specimen, instead of the recommended dry specimens Atterberg limits tests were started with fully-saturated specimens. The Atterberg limits specimen was derived from the combined trimmings from each Shelby tube sample. Hydrometer analyses were conducted according to the provisions of ASTM D422 (2007): Standard Test Method for Particle-Size Analysis of Soils on the combined trimmings from some Shelby tube samples.

Two types of consolidation tests, the Incremental Loading (IL) test and the Constant Rate of Strain (CRS) test, were conducted as part of the TGRP. The IL test was carried out at S&T while the CRS test was carried out at MU. The IL consolidation tests were conducted according to the provisions of ASTM D2435 (2004): Standard Test Method for One-Dimensional Consolidation Properties of Soils while the CRS consolidation tests were conducted according to the provisions of ASTM D4186 (2006): Standard Test Method for One-Dimensional Consolidation Properties of Saturated Cohesive Soils Using Controlled-Strain Loading. The tests were carried at the rate of about one test per Shelby tube sample.

The unconsolidated undrained shear strength (UU) test and the consolidated undrained shear strength (CU) test were conducted according to the provisions of ASTM D2850 (2007): Standard Test Method for Unconsolidated-Undrained Triaxial Compression test on Cohesive Soils and ASTM D4767 (2004): Standard Test Method for Consolidated Undrained Triaxial Compression test for Cohesive Soils, respectively. Both UU and CU tests were conducted on 1.4-inch diameter, 2.8-inch high specimens. The tests were carried at the rate of about two and three tests per Shelby tube sample for the UU and the CU tests, respectively. Detailed descriptions of the testing protocols adopted are presented in Appendix A.

A summary of the number of laboratory tests conducted presented in Table 3.2. Table 3.2 presents a summary of the number of laboratory tests conducted by MU/S&T and MoDOT pursuant to the execution of the TGRP and the number of the same tests obtained from MoDOT's database. The tests of interest are natural water content, Atterberg limits, consolidation, unconfined compressive strength (UCS), UU, CU, and direct shear (DS).

The number of natural water content, Atterberg limits, and consolidation tests conducted by MU/S&T (Research) and MoDOT (Sister holes) and the number of these tests from the MoDOT's database are presented. For the consolidation tests, S&T and MoDOT conducted the IL test while MU conducted the CRS test. Additionally, the number of UU and CU tests and the number of UCS and DS tests conducted are presented. MU/S&T conducted UU and CU tests while MoDOT conducted UCS and DS tests.

Table 3.2: Summary of the Laboratory Tests Conducted in the TGRP

Lab/Test	Natural Water Content	Atterberg Limits	Consolidation	UCS	UU	CU	DS
MU/S&T							
MU/S&T	646	464	$78^{CRS}+54^{IL}$		323	179	-
MoDOT (Sister Boreholes)							
MoDOT	312	276	29^{IL}	72	-	-	35
MoDOT (Database)							
MoDOT	624	483	140^{IL}	70	6	24	80
Notes: UCS = Unconfined Compressive Strength; DS = Direct Shear; IL = Incremental Loading; CRS = Constant Rate of Strain; UU = Unconsolidated Undrained Shear Strength; CU = Unconsolidated Undrained Shear Strength							

3.5. MoDOT's DATABASE

More data was obtained from MoDOT's geotechnical database. Geotechnical data from the four sites described in §3.2 above and KCiCON site in North Kansas City, Missouri were obtained from MoDOT's Database. The data from MoDOT's Database covers the time period from 1978 to 2006. Hence, it includes the effect of seasonal variations on the soil parameters. A summary of data from MoDOT’s geotechnical database are also presented in Table 3.2.

4. DATA ANALYSIS

4.1. INTRODUCTION

Analyses of data to obtain the statistics and parameters required for RBD is performed in this section. The methodologies employed in these analyses and their results are presented in this section.

This section is divided into four parts: (a) Second Moment Statistics – in which the mean, variance and the COV of parameters under various conditions are determined; (b) Probability Distribution – in which the probability distribution type of parameters under various conditions are determined and the effect of in-situ state on the probability distribution type of parameters are evaluated; (c) Correlation of Soil Properties – in which correlation matrices of parameters are developed, empirical correlations between parameters are developed, and the validity of published empirical relationships between parameters as they relate to this study are assessed; and (d) Spatial Variability – in which the scale of fluctuation, θ, of parameters are determined, the range of influence computed using laboratory data and CPTu data are compared, and the scale of fluctuation using the autocorrelation function and the semivariogram functions are compared.

4.2. SECOND MOMENT STATISTICS: MEAN, VARIANCE, AND COEFFICIENT OF VARIATION (COV)

The descriptive statistical analysis for this study was carried out using Microsoft Excel 2007 add-in for data analysis. Since it is not possible to exhaustively determine all the realizations of a random variable in a population, statistical moments are usually determined from a sample of the population. The first (mean) and second statistical moments (variance) were determined using Equations 13 and 14, respectively for both laboratory test and field test data. Equation 15 was used to determine the COV of the various parameters.

$$\text{Sample mean}, \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \tag{13}$$

$$\text{Sample variance}, s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} \tag{14}$$

Where n is the total number of data points in a sample, and x_i = is each data point.

$$\text{Coefficient of variation, COV} = \frac{\sqrt{s^2}}{\bar{x}} \tag{15}$$

In cases where there are few data, the second moment statistics were determined following the method in Snedecor and Cochran (1964) as described by Lacasse and Nadim (1996). Snedecor and Cochran (1964) gave weighting factors, presented in Table 4.1, for computing the second moment statistics for datasets having up to and including 20 data points. Notionally, this could mean that statistics computed with data equal to less than or equal to 20 could be deemed defective. In this case, the standard deviation, s (= square root of variance) is determined from:

$$\text{Standard Deviation}, \; s = \text{Range} * \text{Weighting Factor} \tag{16}$$

Table 4.1: Short-Cut Estimates for Limited Dataset (Snedecor and Cochran, 1964) [from Lacasse and Nadim, 1996]

# Points	Weighting Factor	# Points	Weighting Factor	# Points	Weighting Factor	# Points	Weighting Factor
1	-	6	0.395	11	0.315	16	0.283
2	0.886	7	0.370	12	0.307	17	0.279
3	0.591	8	0.351	13	0.300	18	0.275
4	0.486	9	0.337	14	0.294	19	0.271
5	0.430	10	0.325	15	0.288	20	0.268

Studies were led to determine the second moment statistics of geotechnical parameters and investigate the effect of in-situ state (above ground water level, and below ground water level) and soil classification (USCS) type on second moment statistics. The following steps were taken to determine the second moment statistics:

- Data (laboratory and CPT) was collated and validated. Data validation involved the identification and verification of suspected outliers.
- Validated data was used to determine the second moment statistics of geotechnical parameters at both the location and state levels.
- Validated data was grouped according to their in-situ state (above ground water level, and below ground water level) and the second moment statistics of geotechnical parameters was determined at both the location and state levels.
- Validated data grouped according to their in-situ state was grouped further according to their soil classification (USCS) type and the second moment statistics of geotechnical parameters was determined at both the location and state levels.

This procedure follows the method of data analysis applied to geotechnical data by Rethati (1987) and is thought to represent best practice in the statistical characterization of geotechnical parameters (Uzielli, et al., 2007).

The geotechnical parameters of interest in the case of the laboratory data include natural moisture content, Atterberg limits (PL, LL, PI, LI), unit weight (bulk, and dry), undrained shear strength (unconsolidated, undrained), S_u (UU), undrained shear strength (consolidated, undrained), S_u (CU), compression index, Cc, preconsolidation pressure, p'_c, and friction angle, φ' (single test S_u (CU) test, and direct shear test). For the field (CPT) data, the geotechnical parameters of interest include the measured cone tip resistance, q_c, total (corrected) cone resistance, $q_{t,}$ undrained shear strength, S_u, constrained modulus, compression index, Cc, preconsolidation pressure, p_c', friction angle, ϕ', and normalized (with respect to effective overburden pressure) total (corrected) cone resistance, q_t/σ_o'.

The undrained shear strength, S_u from the field (CPTu) data was derived by calibrating the CPTu data the laboratory test data, following one of the methods in Lunne et al., (1997), so that both results are comparable. The calibration of CPTu data against laboratory test data for this study was executed by Dr Norbert Maerz and Kerry Magner both of the Field Testing team of the TGRP. The calibration procedure, which principally aims at obtaining the appropriate cone factor, N_{kt}, for the CPTu data, is presented following.

- Validate both laboratory S_u (UU) data and CPTu data
- Develop both laboratory S_u (UU) profile and CPTu profile
- Fit a model to the laboratory S_u (UU) profile

- On the basis of the fitted model determine laboratory S_u (UU) values at the same spacing as the CPTu data, S_u (Model).
- Using the equation that relates undrained shear strength to corrected cone resistance (Lunne et al., 1997):

$$S_u = \frac{(q_t - \sigma_{vo})}{N_{kt}} \tag{17}$$

where q_t = corrected cone resistance; σ_{vo} = total in-situ overburden stress; and N_{kt} = variable cone factor.

- Rearranging Equation (17):

$$N_{kt} = \frac{(q_t - \sigma_{vo})}{S_u} \tag{18}$$

- Plot a graph of (q_t – σ_{vo}) versus S_u (Model), and plot the best-fit linear relationship between (q_t – σ_{vo}) versus S_u (Model).
- The reciprocal of the slope of best-fit line is the N_{kt} value for the profile to be used in Equation 17 to obtain calibrated S_u values from the CPTu profile.

The bearing factor, N_{kt}, values for soundings at each location are presented in Table 4.2.

The results of the second moment statistics analysis are presented in Tables 4.3 to 4.9. The results presented in Tables 4.3 to 4.9 are the second moment statistics for parameters in Warrensburg that have a data count of five and above. In the aforementioned tables, the terms classified and unclassified are used in two contexts: (a) classification according to in-situ state as either above ground water level (AGWL) or below ground water level (BGWL); and (b) classification of soil types according to the Unified Soil Classification System (USCS). Whereas classification in terms of in-situ state is usually specified in terms of stress history, classification in terms of ground water level was adopted in this study for simplicity and also to investigate the effect of location relative to the ground water level on the statistical properties of geotechnical parameters. The comprehensive results including the results for various locations are presented in Appendices C and D for Laboratory and field data, respectively.

Table 4.2: Bearing Factor, N_{kt}, Values

Location	CPTu Sounding (B)						
	1	2	3	4	5	6	7
Warrensburg (W)	6	13	-	-	-	-	-
St. Charles (SC)	20	24	20	15	-	-	-
New Florence (NF)	20	24	20	15	-	-	-
Pemiscot (P)	15	16	14	15	15	14	16
Notes: W-B1 = Warrensburg Sounding #1; P-B7 = Pemiscot Sounding #7							

The second moment statistics based on the laboratory test data are presented in Tables 4.3 and 4.8. The second moment statistics for Warrensburg: unclassified, unclassified – AGWL, and unclassified – BGWL are presented in Tables 4.3, 4.4, and 4.5, respectively. Similarly, the second moment statistics for Warrensburg: classified, classified – AGWL, and classified – BGWL are presented in Tables 4.6, 4.7, and 4.8, respectively. Tables 4.3 to 4.8 are divided into four subsets: a, b, c, and d. a presents the second moment statistics of moisture content and unit weight, while b, c, and d present the statistics for Atterberg limits, strength parameters and consolidation parameters, respectively.

The second moment statistics based on the field data are presented in Table 4.9. Table 4.9 has three subsets: a, b, and c. a, b, and c presents the second moment statistics of parameters for Warrensburg: classified, classified – AGWL, classified – BGWL, respectively.

The second moment statistics based on both laboratory and field testing are compared to published data, principally data from Uzielli et al., (2007), Corotis et al., (1975) and Loehr et al., (2005) as presented in Tables 4.3 to 4.9. It should be noted that data presented in Uzielli et al., (2007) is a summary of data from other sources like Phoon et al., (1995), Kulhawy and Trautmann (1996), Lacasse and Nadim (1996), Phoon and Kulhawy (1999a), and Jones et al., (2002).

Table 4.3: Lab Data, Second Moment Statistics for Warrensburg (Unclassified)

Parameter / Soil Type	Mean	Variance	COV (%)	Count	Reported COV (%) / Reference
(a) MOISTURE CONTENT AND UNIT WEIGHT					
w_n (%)	26.3	14.8	15	326	8-30/1; 47/2
γ_b (pcf)	119.4	40.7	5	231	<10/1; 15/2
γ_d (pcf)	94.3	41.7	7	231	<10/1; 15/2
(b) ATTERBERG LIMITS					
LL (%)	36.1	74.2	23	291	6-30/1; 34/2
PL (%)	19.2	16.3	21	290	6-30/1; 29/2
PI (%)	17.8	58.3	43	290	48/2
LI	0.49	0.07	54	186	34/2

Table 4.3 Cont'd: Lab Data, Second Moment Statistics for Warrensburg (Unclassified)

Parameter / Soil Type	Mean	Variance	COV (%)	Count	Reported COV (%) / Reference
(c) STRENGTH PARAMETERS					
S_u [UU] (psf)	1534.6	2.1E+06	95	68	10-30/1
$Q_u/2$ (psf)	1157.5	5.8E+05	66	20	20-55/1
S_{up} [CU] (psf)	2960.3	3.7E+06	65	42	20-40/1
φ_p [CU] (deg)	26.3	61.5	30	42	5-15/1
φ [DS] (deg)	29	19.6	15	21	
$\varphi_{C=0}$ [DS] (deg)	31	11.4	11	21	
C_u [DS] (psf)	203.9	13020.3	56	21	
(d) CONSOLIDATION PARAMETERS					
e_0 [IL+CRS]	0.8	0.01	15	231	7-30/1; 41.5/2
Cc [IL+CRS]	0.231	0.029	74	53	10-37/1; 73/2
Cr [IL+CRS]	0.037	0.002	114	52	
$p'_{c\text{-}T}$ (psf) [IL+CRS]	2186.1	1.5E+07	57	28	10-35/1; 62/2
$p'_{c\text{-}C}$ (psf) [IL+CRS]	2488.4	1.6E+07	51	52	10-35/1; 62/2
$p'_{c\text{-}SE}$ (psf) [IL+CRS]	2708.4	1.6E+07	46	29	10-35/1; 62/2

Notes: 1= Uzielli et al., (2007); 2 = Corotis et al., (1975); w_n = natural water content; γ_b = bulk unit weight; γ_d = dry unit weight; LL = liquid limit; PL = plastic limit; PI = plasticity index; LI = liquidity index; S_u (UU) = undrained shear strength (unconsolidated, undrained); $Q_u/2$ = unconfined compressive strength; S_{up} (CU) = peak undrained shear strength (consolidated, undrained); φ_P = friction angle (peak); φ [DS] = friction angle [Direct Shear]; $\varphi_{C=0}$ [DS] = friction angle @ C_u = 0 [Direct Shear]; C_u = cohesion; e_0 = initial void ratio; Cc = compression index; Cr = recompression index; $p'_{c\text{-}T}$ = preconsolidation pressure (Tangent); $p'_{c\text{-}C}$ = preconsolidation pressure (Casagrande); $p'_{c\text{-}SE}$ = preconsolidation pressure (Strain Energy)

Table 4.4: Lab Data, Second Moment Statistics for Warrensburg (Unclassified, In-Situ State: AGWL)

Parameter / Soil Type	Mean	Variance	COV (%)	Count	Reported COV (%) / Reference
(a) MOISTURE CONTENT AND UNIT WEIGHT					
w_n (%)	23.3	19.6	19	64	8-30/1
γ_b (pcf)	120.9	53.2	6	30	<10/1
γ_d (pcf)	96.1	52.8	8	30	<10/1; 15/3

Table 4.4 Cont'd: Lab Data, Second Moment Statistics for Warrensburg (Unclassified, In-Situ State: AGWL)

Parameter / Soil Type	Mean	Variance	COV (%)	Count	Reported COV (%) / Reference
(b) ATTERBERG LIMITS					
LL (%)	40.1	66.1	20	53	6-30/1; 34/2
PL (%)	20.5	11.3	16	53	6-30/1; 29/2
PI (%)	19.6	80.1	46	53	48/2
LI	0.32	0.062	78	19	34/2
(c) STRENGTH PARAMETERS					
$Q_u/2$ (psf)	1545.3	601188.2	50	10	20-55/1
φ [DS] (deg)	26.7	23.9	18	7	
$\varphi_{C=0}$ [DS] (deg)	29.0	16.5	14	7	
C_u [DS] (psf)	244.6	14808.9	50	7	
(d) CONSOLIDATION PARAMETERS					
e_0 [IL+CRS]	0.8	0.02	18	30	7-30/1; 41.5/2
Cc [IL+CRS]	53	0.2	0.001	16	10-37/1; 73/2
Cr [IL+CRS]	52	0.04	0.0002	34	
$p'_{c\text{-}C}$ (psf) [IL+CRS]	52	2063.3	1.4E+06	57	10-35/1; 62/2

Notes: 1= Uzielli et al., (2007); 2 = Corotis et al., (1975); 3 = Loehr et al., (2005); w_n = natural water content; γ_b = bulk unit weight; γ_d = dry unit weight; LL = liquid limit; PL = plastic limit; PI = plasticity index; LI = liquidity index; $Q_u/2$ = unconfined compressive strength; φ [DS] = friction angle [Direct Shear]; $\varphi_{C=0}$ [DS] = friction angle @ C = 0 [Direct Shear]; C_u = Cohesion; e_0 = initial void ratio; Cc = compression index; Cr = recompression index; $p'_{c\text{-}C}$ = preconsolidation pressure (Casagrande)

Table 4.5: Lab Data, Second Moment Statistics for Warrensburg (Unclassified, In-Situ State: BGWL)

Parameter / Soil Type	Mean	Variance	COV (%)	Count	Reported COV (%) / Reference
(a) MOISTURE CONTENT AND UNIT WEIGHT					
w_n (%)	27.3	9.4	11	248	8-30/1
γ_b(pcf)	119.0	38.8	5	185	<10/1
γ_d (pcf)	93.4	33.9	6	185	<10/1; 15/3
(b) ATTERBERG LIMITS					
LL (%)	34.7	35.2	17	224	6-30/1; 34/2
PL (%)	18.5	13.2	20	223	6-30/1; 29/2

Table 4.5 Cont'd: Lab Data, Second Moment Statistics for Warrensburg (Unclassified, In-Situ State: BGWL)

Parameter / Soil Type	Mean	Variance	COV (%)	Count	Reported COV (%) / Reference
PI (%)	16.3	29.2	33	223	48/2
LI	0.52	0.06	49	161	34/2
(c) STRENGTH PARAMETERS					
S_u [UU] (psf)	1310.1	8.5E+05	70	51	10-30/1
$Q_u/2$ (psf)	769.6	2.9E+05	70	10	20-55/1
S_{up} [CU] (psf)	2958.6	3.8E+06	66	41	20-40/1
φ_p [CU] (deg)	26.4	62.4	30	41	5-15/1
(d) CONSOLIDATION PARAMETERS					
e_0 [IL+CRS]	0.8	0.01	14	185	7-30/1; 41.5/2
Cc [IL+CRS]	0.236	0.032	76	48	10-37/1; 73/2
Cr [IL+CRS]	0.037	0.002	119	47	
$p'_{c\text{-}T}$ (psf) [IL+CRS]	2227.4	1.5E+06	56	27	10-35/1; 62/2
$p'_{c\text{-}C}$ (psf) [IL+CRS]	2523.8	1.7E+06	51	47	10-35/1; 62/2
$p'_{c\text{-}SE}$ (psf) [IL+CRS]	2745.0	1.6E+06	46	28	10-35/1; 62/2

Notes: 1 = Uzielli et al., (2007); 2 = Corotis et al., (1975); 3 = Loehr et al., (2005); w_n = natural water content; γ_b = bulk unit weight; γ_d = dry unit weight; LL = liquid limit; PL = plastic limit; PI = plasticity index; LI = liquidity index; S_u (UU) = undrained shear strength (unconsolidated, undrained); $Q_u/2$ = unconfined compressive strength; S_{up} (CU) = peak undrained shear strength (consolidated, undrained); φ_P = friction angle (peak); e_0 = initial void ratio; Cc = compression index; Cr = recompression index; $p'_{c\text{-}T}$ = preconsolidation pressure (Tangent); $p'_{c\text{-}C}$ = preconsolidation pressure (Casagrande); $p'_{c\text{-}SE}$ = preconsolidation pressure (Strain Energy)

Table 4.6: Lab Data, Second Moment Statistics for Warrensburg (Classified)

Parameter / Soil Type	Mean	Variance	COV (%)	Count	Reported COV (%) / Reference
(a) MOISTURE CONTENT AND UNIT WEIGHT					
Natural Moisture Content, w_n (%)					
CL	26.7	13.4	14	297	8-30/1; 22/3
CH	21.7	24.5	23	8	8-30/1; 49/3
Bulk Unit Weight, γ_b (pcf)					
CL	119.2	41.2	5	207	<10/1; 3/3
CH	124.2	7.1	2	5	<10/1; 3/3

Table 4.6 Cont'd: Lab Data, Second Moment Statistics for Warrensburg (Classified)

Parameter / Soil Type	Mean	Variance	COV (%)	Count	Reported COV (%) / Reference
Dry Unit Weight, γ_d(pcf)					
CL	93.7	37.6	7	207	<10/1; 15/3
CH	98.4	12.5	4	5	<10/1; 15/3
(b) ATTERBERG LIMITS					
Liquid Limit, LL (%)					
CL	35.4	32.6	16	258	6-30/1; 34/2
CH	54.7	0.5	1	9	6-30/1; 34/2
Plastic Limit, PL (%)					
CL	18.6	12.1	19	257	6-30/1; 29/2
CH	22.3	9.2	14	9	6-30/1; 29/2
Plasticity Index, PI (%)					
CL	16.9	27.9	31	257	48/2; 33/3
CH	32.3	5.6	7	9	48/2; 19/3
Liquidity Index, LI					
CL	0.50	0.07	51	178	34/2
(c) STRENGTH PARAMETERS					
Undrained Shear Strength, S_u [UU] (psf)					
CL	1310.1	8.5E+05	70	51	10-30/1
Unconfined Compressive Strength, $Q_u/2$ (psf)					
CL	1050.4	6.8E+05	79	16	20-55/1; 80/3
Undrained Shear Strength (peak), S_{up} [CU] (psf)					
CL	2960.3	3.7E+06	65	42	20-40/1
Friction Angle (peak), φ_p [CU] (deg)					
CL	26.3	61.5	30	42	5-15/1
Friction Angle, φ [DS] (deg)					
CL	28.7	20.7	16	19	19/3
Friction Angle (@ $C_u = 0$), $\varphi_{C=0}$ [DS] (deg)					
CL	30.8	12.2	11	19	16/3
Cohesion, C_u [DS] (psf)					
CL	199.2	10382.8	51	19	76/3
(d) CONSOLIDATION PARAMETERS					
Initial Void Ratio, e_0					
CL	0.8	0.014	14	207	7-30/1; 41.5/2
CH	0.8	0.004	8	5	7-30/1; 41.5/2

Table 4.6 Cont'd: Lab Data, Second Moment Statistics for Warrensburg (Classified)

Parameter / Soil Type	Mean	Variance	COV (%)	Count	Reported COV (%) / Reference
Compression Index, Cc					
CL	0.232	0.030	75	52	10-37/1; 73/2
Recompression Index, Cr					
CL	0.038	0.002	114	51	
Preconsolidation Pressure (Tangent) $p'_{c\text{-}T}$ (psf)					
CL	2186.1	1530115.5	57	28	10-35/1; 62/2
Preconsolidation Pressure (Casagrande) $p'_{c\text{-}C}$ (psf)					
CL	2507.8	1646730.1	51	51	10-35/1; 62/2
Preconsolidation Pressure (Strain Energy) $p'_{c\text{-}SE}$ (psf)					
CL	2708.4	1580306.2	46	29	10-35/1; 62/2
Notes: 1 = Uzielli et al., (2007); 2 = Corotis et al., (1975); 3 = Loehr et al., (2005)					

Table 4.7: Lab Data, Second Moment Statistics for Warrensburg (Classified, In-Situ State: AGWL)

Parameter / Soil Type	Mean	Variance	COV (%)	Count	Reported COV (%) / Reference
(a) MOISTURE CONTENT AND UNIT WEIGHT					
Natural Moisture Content, w_n (%)					
CL	23.5	20.0	19	52	8-30/1; 22/3
CH	21.7	24.5	23	8	8-30/1; 49/3
Bulk Unit Weight, γ_b (pcf)					
CL	120.9	59.3	6	23	<10/1; 3/3
CH	124.2	7.1	2	5	<10/1; 3/3
Dry Unit Weight, γ_d(pcf)					
CL	96.1	62.7	8	23	<10/1; 15/3
CH	98.4	12.5	4	5	<10/1; 15/3
(b) ATTERBERG LIMITS					
Liquid Limit, LL (%)					
CL	38.8	22.3	12	39	6-30/1; 34/2
CH	54.8	0.5	1	8	6-30/1; 34/2

Table 4.7 Cont'd: Lab Data, Second Moment Statistics for Warrensburg (Classified, In-Situ State: AGWL)

Parameter / Soil Type	Mean	Variance	COV (%)	Count	Reported COV (%) / Reference
Plastic Limit, PL (%)					
CL	19.2	3.8	10	39	6-30/1; 29/2
CH	22.6	10.0	14	8	6-30/1; 29/2
Plasticity Index, PI (%)					
CL	19.6	31.0	28	39	48/2; 33/3
CH	32.1	6.0	8	8	48/2; 19/3
Liquidity Index, LI					
CL	0.35	0.066	74	17	34/2
(c) STRENGTH PARAMETERS					
Unconfined Compressive Strength, $Q_u/2$ (psf)					
CL	1373.6	909848.9	69	7	20-55/1; 80/3
Friction Angle, φ [DS] (deg)					
CL	25.0	21.2	18	5	19/3
Friction Angle (@ $C_u = 0$), $\varphi_{C=0}$ [DS] (deg)					
CL	27.4	13.7	14	5	16/3
Cohesion, C_u [DS] (psf)					
CL	243.2	16641.0	53	5	76/3
(d) CONSOLIDATION PARAMETERS					
Initial Void Ratio, e_0					
CL	0.18	0.001	16	23	7-30/1; 41.5/2
CH	0.8	0.004	8	5	7-30/1; 41.5/2
Compression Index, Cc					
CL	0.175	0.001	16	5	10-37/1; 73/2
Recompression Index, Cr					
CL	0.037	0.0002	41	5	
Preconsolidation Pressure (Casagrande) $p'_{c\text{-}C}$ (psf)					
CL	2256.0	1.8E+06	59	5	10-35/1; 62/2
Notes: 1= Uzielli et al., (2007); 2 = Corotis et al., (1975); 3 = Loehr et al., (2005); UU = Unconsolidated, Undrained shear strength; CU = Consolidated, Undrained shear strength; DS = Direct Shear					

Table 4.8: Lab Data, Second Moment Statistics for Warrensburg (Classified, In-Situ State: BGWL)

Parameter / Soil Type	Mean	Variance	COV (%)	Count	Reported COV (%) / Reference
(a) MOISTURE CONTENT AND UNIT WEIGHT					
Natural Moisture Content, w_n (%)					
CL	27.3	9.4	11	248	8-30/1; 22/3
Bulk Unit Weight, γ_b (pcf)					
CL	119.0	38.8	5	185	<10/1; 3/3
Dry Unit Weight, γ_d (pcf)					
CL	93.4	33.9	6	185	<10/1; 15/3
(b) ATTERBERG LIMITS					
Liquidity Index, LL (%)					
CL	34.7	35.2	17	224	6-30/1; 34/2
Plastic Limit, PL (%)					
CL	18.5	13.2	20	223	6-30/1; 29/2
Plasticity Index, PI (%)					
CL	16.3	29.2	33	223	48/2; 33/3
Liquidity Index, LI					
CL	0.52	0.07	49	156	34/2
(c) STRENGTH PARAMETERS					
Undrained Shear Strength, S_u [UU] (psf)					
CL	1310.1	8.5E+05	70	51	10-30/1
Unconfined Compressive Strength, $Q_u/2$ (psf)					
CL	769.6	2.1E+05	59	10	20-55/1; 80/3
Undrained Shear Strength (peak), S_{up} [CU] (psf)					
CL	2958.6	3.8E+06	66	41	20-40/1
Friction Angle (peak), φ_p [CU] (deg)					
CL	26.4	62.4	30	41	5-15/1
Friction Angle, φ [DS] (deg)					
CL	30.1	13.5	12	14	19/3
Friction Angle (@ $C_u = 0$), $\varphi_{C=0}$ [DS] (deg)					
CL	32.0	5.5	7	14	16/3
Cohesion, C_u [DS] (psf)					
CL	183.5	11452.4	58	14	76/3
(d) CONSOLIDATION PARAMETERS					
Initial Void Ratio, e_0					
CL	0.8	0.01	14	185	7-30/1; 41.5/2

Table 4.8 Cont'd: Lab Data, Second Moment Statistics for Warrensburg (Classified, In-Situ State: BGWL

Parameter / Soil Type	Mean	Variance	COV (%)	Count	Reported COV (%) / Reference
Compression Index, Cc					
CL	0.037	0.002	119	47	10-37/1; 73/2
Recompression Index, Cr					
CL	0.236	0.032	76	48	
Preconsolidation Pressure $p'_{c\text{-}T}$ (psf)					
CL	2227.4	1.5E+06	56	27	10-35/1; 62/2
Preconsolidation Pressure $p'_{c\text{-}C}$ (psf)					
CL	2523.8	1.7E+06	51	47	10-35/1; 62/2
Preconsolidation Pressure $p'_{c\text{-}SE}$ (psf)					
CL	2745.0	1.6E+06	46	28	10-35/1; 62/2
Notes: 1= Uzielli et al., (2007); 3 = Loehr et al., (2005); UU = Unconsolidated, Undrained shear strength; CU = Consolidated, Undrained shear strength; DS = Direct Shear;					

Table 4.9: CPTu Data, Second Moment Statistics for Warrensburg (Classified)

Soil Property / Soil Type	Mean	Variance	COV (%)	Count	Reported COV (%) / Reference
(a) CLASSIFIED					
Cone Resistance, q_c (tsf)					
1. Sensitive Fine-Grained	4.2	0.1	6.0	6	
2. Organic Soil	3.6	3.5	52.2	58	
3. Clay	7.6	15.1	51.0	629	20-40/1
4. Clay & Silty Clay	27.6	352.4	68.1	44	
5. Silty Sand & Sandy Silt	64.7	2115.6	71.1	36	
9. Very Dense/Stiff Soil^{+}	9.5	22.4	49.6	22	
Side Friction, f_s (tsf)					
1. Sensitive Fine-Grained	0.015	0.0001	52.7	6	
2. Organic Soil	0.3	0.03	61.4	58	
3. Clay	0.4	0.1	73.0	629	
4. Clay & Silty Clay	0.6	0.5	107.8	44	
5. Silty Sand & Sandy Silt	1.2	1.9	118.8	36	
9. Very Dense/Stiff Soil^{+}	0.6	0.1	51.4	22	

Table 4.9 Cont'd: CPTu Data, Second Moment Statistics for Warrensburg (Classified)

Corrected Cone Resistance, q_t (tsf)					
1. Sensitive Fine-Grained	4.7	0.1	5.8	6	
2. Organic Soil	3.8	3.8	51.5	58	
3. Clay	7.9	14.6	48.5	629	< 20/1
4. Clay & Silty Clay	27.5	332.1	66.3	44	
5. Silty Sand & Sandy Silt	64.3	2099.6	71.2	36	
9. Very Dense/Stiff Soil^{+}	9.5	22.2	49.5	22	
Undrained Shear Strength, S_u (psf)					
1. Sensitive Fine-Grained	3.81E+02	1.81E+03	11.2	6	
2. Organic Soil	5.03E+02	1.56E+05	78.6	33	
3. Clay	1.67E+03	2.02E+06	84.9	493	
4. Clay & Silty Clay	6.90E+03	3.03E+07	79.8	37	
9. Very Dense/Stiff Soil^{+}	3.25E+03	1.88E+06	42.28	18	
Normalized Corrected Cone Resistance, q_t/σ'_{vo}					
1. Sensitive Fine-Grained	5.03E+03	3.06E+05	11.0	6	
2. Organic Soil	4.71E+03	4.20E+06	43.5	33	
3. Clay	1.27E+04	5.89E+07	60.6	493	
4. Clay & Silty Clay	5.27E+04	1.32E+09	68.9	37	
5. Silty Sand & Sandy Silt	1.21E+05	6.23E+09	65.2	36	
9. Very Dense/Stiff Soil^{+}	2.14E+04	4.48E+07	31.3	18	
(b) CLASSIFIED, IN-SITU STATE: AGWL					
Cone Resistance, q_c (tsf)					
2. Organic Soil	1.8	0.6	43.8	14	20-40/1
3. Clay	5.0	7.4	54.0	90	
9. Very Dense/Stiff Soil^{+}	9.5	22.4	49.6	22	
Side Friction, f_s (tsf)					
2. Organic Soil	0.2	0.005	32.9	14	
3. Clay	0.4	0.1	64.0	90	
9. Very Dense/Stiff Soil^{+}	0.6	0.1	51.4	22	
Corrected Cone Resistance, q_t (tsf)					
2. Organic Soil	1.8	0.5	40.0	14	< 20/1
3. Clay	5.1	7.1	52.6	90	
9. Very Dense/Stiff Soil^{+}	9.5	22.2	49.5	22	
Undrained Shear Strength, S_u (psf)					
2. Organic Soil	5.69E+02	2.25E+04	26.4	8	
3. Clay	1.08E+03	2.92E+05	49.9	67	
9. Very Dense/Stiff Soil^{+}	3.25E+03	1.88E+06	42.3	18	
Normalized Corrected Cone Resistance, q_t/σ'_{vo}					
2. Organic Soil	4.42E+03	5.03E+05	16.1	8	
3. Clay	1.04E+04	3.32E+07	55.4	67	

Table 4.9 Cont'd: CPTu Data, Second Moment Statistics for Warrensburg (Classified)

Soil Property / Soil Type	Mean	Variance	COV (%)	Count	Reported COV (%) / Reference
9. Very Dense/Stiff Soil[+]	2.14E+04	4.48E+07	31.3	18	
(c) CLASSIFIED, IN-SITU STATE: BGWL					
Cone Resistance, q_c (tsf)					
1. Sensitive Fine-Grained	4.2	0.1	6.0	6	
2. Organic Soil	4.2	3.0	41.6	44	
3. Clay	8.8	42.4	74.1	551	20-40/1
4. Clay & Silty Clay	27.6	352.4	68.1	44	
5. Silty Sand & Sandy Silt	64.7	2115.6	71.1	36	
Side Friction, f_s (tsf)					
1. Sensitive Fine-Grained	0.015	0.0001	52.7	6	
2. Organic Soil	0.3	0.0	63.9	44	
3. Clay	0.4	0.1	74.7	551	
4. Clay & Silty Clay	0.4	0.1	74.7	551	
5. Silty Sand & Sandy Silt	1.2	1.9	118.8	36	
Corrected Cone Resistance, q_t (tsf)					
1. Sensitive Fine-Grained	4.7	0.1	5.8	6	
2. Organic Soil	4.4	3.1	40.1	44	
3. Clay	9.1	41.3	70.8	551	< 20/1
4. Clay & Silty Clay	27.5	332.1	66.3	44	
5. Silty Sand & Sandy Silt	64.3	2099.6	71.2	36	20-40/1
Undrained Shear Strength, S_u (psf)					
1. Sensitive Fine-Grained	3.81E+02	1.81E+03	11.2	6	
2. Organic Soil	4.82E+02	1.98E+05	92.2	25	
3. Clay	1.92E+03	3.74E+06	100.8	438	
4. Clay & Silty Clay	6.90E+03	3.03E+07	79.8	37	
Normalized Corrected Cone Resistance, q_t/σ'_{vo}					
1. Sensitive Fine-Grained	5.03E+03	3.06E+05	11.0	6	
2. Organic Soil	4.80E+03	5.44E+06	48.6	25	
3. Clay	1.48E+04	1.83E+08	91.6	438	
4. Clay & Silty Clay	5.27E+04	1.32E+09	68.9	37	
5. Silty Sand & Sandy Silt	1.21E+05	6.23E+09	65.2	36	
Notes: 1= Uzielli et al., (2007); σ'_{vo} = effective overburden pressure); σ'_{vo} = effective overburden pressure					

4.3. PROBABILITY DISTRIBUTION

In this study, the Pearson's moment-based technique was adopted for the selection of the probability distribution type that best approximates a dataset. In the Pearson's moment-based technique a suitable probability distribution is identified on the Pearson space based on third- and fourth-moment statistics, i.e. skewness, C_s and kurtosis, C_k of the dataset. C_s and C_k are respectively computed using Equations 19 and 20 below.

$$\text{Skewness, } C_s = \frac{n\sum_{i=1}^{n}(x_i - \bar{x})^3}{(n-1)(n-2)s^3} \tag{19}$$

$$\text{Kurtosis, } C_k = \frac{n\sum_{i=1}^{n}(x_i - \bar{x})^4}{(n-1)(n-2)s^4} - 3 \tag{20}$$

Where n is the total number of data points in a sample, x_i is each data point, s is the standard deviation of the dataset, and $\bar{x}$ is mean of the dataset.

The abscissa and ordinate of the Pearson's distribution space are given by Rethati (1988) as:

$$\beta_1 = C_s^2 \tag{21}$$

$$\beta_2 = C_k + 3 \tag{22}$$

With β_1 and β_2 determined, the appropriate probability distribution for a parameter is chosen from the Pearson space.

Following from §4.2 (Second Moment Statistics), studies were led to determine the probability distribution of geotechnical parameters and investigate the effect of in-situ state (above Ground Water Level, and below Ground Water Level) and soil classification (USCS) type on probability distribution. In each case, the analysis for the second moment statistics (§4.2) was extended by using the mean and variance to compute C_s and C_k. C_s and C_k were used to determine β_1 and β_2, respectively. A probability distribution is then chosen on the basis of the location of β_1 and β_2 on the Pearson space.

A sample data for the determination of the probability distribution of undrained strength (UU) and corrected tip resistance for Statewide (data from all locations) [unclassified] condition is presented in Table 4.10 while the location of the data on the Pearson space is presented in Figure 4.1. It should be noted that the abscissa and the ordinate of the Pearson space can be extended to the hundreds. Extended versions of the Pearson space, adapted from (Pearson, 1956; Pearson and Hartley, 1976; Pearson and Hartley, 1972), with abscissa and ordinate up to 250 are presented in Appendix E.

4.3.1. Results. The results of the probability distribution analyses are presented in Tables 4.11 to 4.17. The results presented in Tables 4.11 to 4.17 are the probability distribution for parameters in Warrensburg that have a data count of five and above. In the aforementioned tables, the terms classified and unclassified are used in two contexts: (a) classification according to in-situ state as either above ground water level (AGWL) or below ground water level (BGWL); and (b) classification of soil types according to the Unified Soil Classification System (USCS). The comprehensive results including the results for various locations are presented in Appendices F and G for laboratory and field data, respectively.

Table 4.10: Sample Data for the Determination of Probability Distribution of Undrained Shear Strength (UU) and Corrected Tip Resistance, q_t, for Statewide (Unclassified) Condition

Identifier	Count	β_1	β_2	Pearson Type
Laboratory				
S_u (UU)	329	6.2	15.7	VI
S_u (UU) / AGWL	104	0.1	2.3	I(∩)
S_u (UU) / BGWL	207	2.6	5.8	I(J)
CPTu - Clay				
q_t	4304	6.4	11.3	I(J)
q_t / AGWL	323	0.4	4.4	IV
q_t / BGWL	3993	7.3	13.1	I(J)
Notes: AGWL = above groundwater level; BGWL = below ground water level				

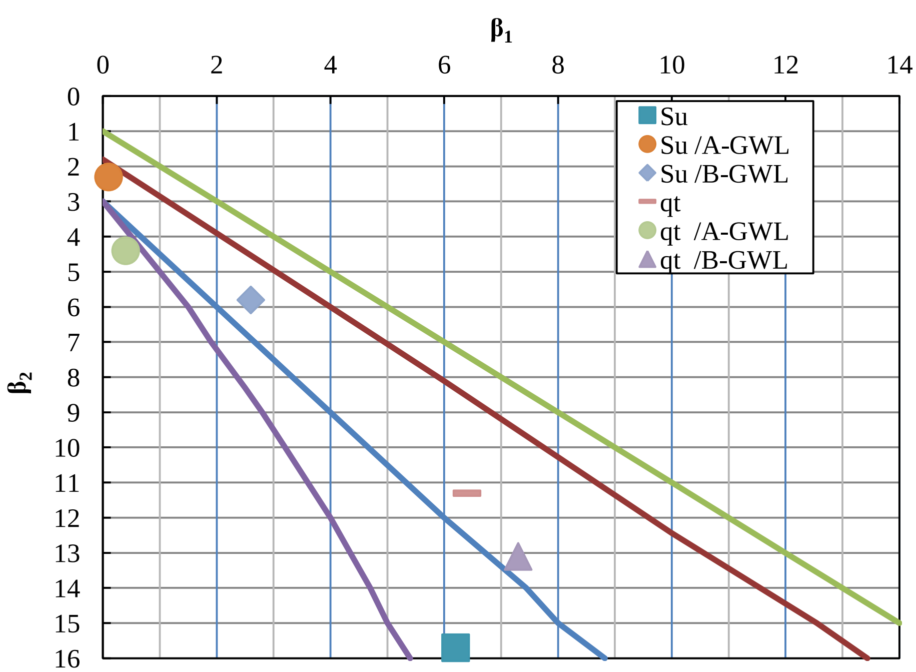


Figure 4.1: Location of S_u (UU) and q_t (for Clay) on the Pearson Space for the Unclassified, AGWL and BGWL Conditions

The probability distribution based on the validated laboratory test data are presented in Tables 4.11 and 4.16. The probability distribution for Warrensburg: unclassified, unclassified – AGWL, and unclassified – BGWL are presented in Tables 4.11, 4.12, and 4.13, respectively. Similarly, the probability distribution for Warrensburg: classified, classified – AGWL, and classified – BGWL are presented in Tables 4.14, 4.15, and 4.16, respectively. Tables 4.11 to 4.16 are divided into four subsets: *a*, *b*, *c*, and *d*. *a* presents the second moment statistics of moisture content and unit weight, while *b*, *c*, and *d* present the statistics for Atterberg limits, strength parameters and consolidation parameters, respectively.

The probability distribution based on the field data are presented in Table 4.17. Table 4.17 has three subsets: *a*, *b*, and *c*. *a*, *b*, and *c* presents the second moment statistics of parameters for Warrensburg: classified, classified – AGWL, classified – BGWL, respectively.

The probability distribution based on both laboratory and field testing are compared to published data, principally data from Uzielli et al., (2007), Corotis et al., (1975) and Loehr et al., (2005) as presented in Tables 4.11 to 4.17. It should be noted that data presented in Uzielli et al., (2007) is a summary of data from other sources like Phoon et al., (1995), Kulhawy and Trautmann (1996), Lacasse and Nadim (1996), Phoon and Kulhawy (1999a), and Jones et al., (2002). Also, in Tables 4.10 to 4.16, with respect to Reported Distribution, N means Normal while LN means Lognormal.

Table 4.11: Lab Data, Probability Distribution for Warrensburg (Unclassified)

Parameter	Count	β_1	β_2	Pearson Type	Reported Distribution / References
(a) MOISTURE CONTENT AND UNIT WEIGHT					
w_n (%)	326	0.1	4.2	IV	Beta/2
γ_b (pcf)	231	0.2	3.4	IV	N/1
γ_d (pcf)	231	0.0	3.2	VII	N/1
(b) ATTERBERG LIMITS					
LL (%)	291	3.0	8.5	VI	N/1,2
PL (%)	290	0.2	6.2	IV	N/1;LN/2
PI (%)	290	1.4	6.4	IV	
LI	186	0.05	2.03	I(∩)	
(c) STRENGTH PARAMETERS					
S_u [UU] (psf)	68	19.5	30.8	I(J)	N,LN/1
$Q_u/2$ (psf)	20	0.8	3.9	I(∩)	
S_{up} [CU] (psf)	42	0.6	2.7	I(J)	N,LN/1
φ_p [CU] (deg)	42	0.3	2.4	I(∩)	
φ [DS] (deg)	21	0.3	2.9	I(∩)	
$\varphi_{C=0}$ [DS] (deg)	21	0.6	3.4	I(∩)	
C_u [DS] (psf)	21	0.1	2.2	I(∩)	
(d) CONSOLIDATION PARAMETERS					
e_0	231	0.2	3.5	IV	N/1; LN/2
Cc	53	41.2	47.6	I (J)	Gamma/2
Cr	52	35.9	43.3	I (J)	
$p'_{c\text{-}T}$ (psf)	28	0.01	2.1	I(∩)	LN/1
$p'_{c\text{-}C}$ (psf)	52	0.6	3.7	IV	LN/1
$p'_{c\text{-}SE}$ (psf)	29	0.06	3.0	IV	LN/1

Notes: 1 = Uzielli et al., (2007); 2 = Corotis et al., (1975)); 3 = Loehr et al., (2005); w_n = natural water content; γ_b = bulk unit weight; γ_d = dry unit weight; LL = liquid limit; PL = plastic limit; PI = plasticity index; LI = liquidity index; S_u (UU) = undrained shear strength (unconsolidated, undrained); $Q_u/2$ = unconfined compressive strength; S_{up} (CU) = peak undrained shear strength (consolidated, undrained); φ_P = friction angle (peak); φ [DS] = friction angle [Direct Shear]; $\varphi_{C=0}$ [DS] = friction angle @ C_u = 0 [Direct Shear]; C_u = Cohesion;e_0 = initial Void Ratio; Cc = compression index; Cr = recompression index; $p'_{c\text{-}T}$ = preconsolidation pressure (Tangent); $p'_{c\text{-}C}$ = preconsolidation pressure (Casagrande); $p'_{c\text{-}SE}$ = preconsolidation pressure (Strain Energy)

Table 4.12: Lab Data, Probability Distribution for Warrensburg (Unclassified, In-Situ State: AGWL)

Parameter	Count	β_1	β_2	Pearson Type	Reported Distribution / Reference
(a) MOISTURE CONTENT AND UNIT WEIGHT					
w_n (%)	64	0.04	4.0	IV	Beta/2
γ_b (pcf)	30	1.4	4.1	I(J)	N/1
γ_d (pcf)	30	0.6	3.9	III	N/1
(b) ATTERBERG LIMITS					
LL (%)	53	0.2	2.6	I(∩)	N/1,2
PL (%)	53	0.8	3.7	I(∩)	N/1;LN/2
PI (%)	53	0.1	2.6	I(∩)	
LI	19	1.09	4.02	I(∩)	
(c) STRENGTH PARAMETERS					
$Q_u/2$ (psf)	10	1.3	4.1	I(∩)	
φ [DS] (deg)	7	0.6	1.9	I (U)	
$\varphi_{C=0}$ [DS] (deg)	7	0.2	1.9	I (U)	
C_u [DS] (psf)	7	0.02	1.8	I (J)	
(d) CONSOLIDATION PARAMETERS					
e_0	30	1.7	5.1	I(∩)	N/1; LN/2
Cc	16	0.3	2.7	I(∩)	Gamma/2
Cr	34	0.6	4.1	VI	
$p'_{c\text{-}C}$ (psf)	57	3.4	6.8	I (J)	LN/1

Notes: 1 = Uzielli et al., (2007); 2 = Corotis et al., (1975)); w_n = natural water content; γ_b = bulk unit weight; γ_d = dry unit weight; LL = liquid limit; PL = plastic limit; PI = plasticity index; LI = liquidity index; $Q_u/2$ = unconfined compressive strength; φ [DS] = friction angle [Direct Shear]; $\varphi_{C=0}$ [DS] = friction angle @ $C_u = 0$ [Direct Shear]; C_u = Cohesion; e_0 = initial void ratio; Cc = compression index; Cr = recompression index; $p'_{c\text{-}T}$ = preconsolidation pressure (Tangent); $p'_{c\text{-}C}$ = preconsolidation pressure (Casagrande); $p'_{c\text{-}SE}$ = preconsolidation pressure (Strain Energy)

Table 4.13: Lab Data, Probability Distribution for Warrensburg (Unclassified, In-Situ State: BGWL)

Parameter	Count	β_1	β_2	Pearson Type	Reported Distribution / Reference
(a) MOISTURE CONTENT AND UNIT WEIGHT					
w_n (%)	248	0.1	4.5	IV	Beta/2

Table 4.13 Cont'd: Lab Data, Probability Distribution for Warrensburg

Parameter	Count	β_1	β_2	Pearson Type	Reported Distribution / Reference
γ_b (pcf)	185	0.2	3.3	III	N/1
γ_d (pcf)	185	0.03	3.3	IV	N/1
(b) ATTERBERG LIMITS					
LL (%)	224	0.3	4.0	IV	N/1,2
PL (%)	223	2.2	7.0	VI	N/1; LN/2
PI (%)	223	0.2	4.0	IV	
LI	161	0.03	2.00	I(∩)	
(c) STRENGTH PARAMETERS					
S_u [UU] (psf)	51	0.7	2.8	I(J)	N,LN/1
$Q_u/2$ (psf)	10	0.3	1.6	I (U)	
S_{up} [CU] (psf)	41	0.6	2.7	I(J)	N,LN/1
φ_p [CU] (deg)	41	0.3	2.4	I(∩)	
(d) CONSOLIDATION PARAMETERS					
e_0	185	0.1	3.2	V	N/1; LN/2
Cc	48	37.8	43.8	I(J)	Gamma/2
Cr	47	33.4	40.0	I(J)	
$p'_{c\text{-}T}$ (psf)	27	0.0	2.2	II	LN/1
$p'_{c\text{-}C}$ (psf)	47	0.5	3.8	IV	LN/1
$p'_{c\text{-}SE}$ (psf)	28	0.04	3.0	I(∩)	LN/1

Notes: 1 = Uzielli et al., (2007); 2 = Corotis et al., (1975); 3 = Loehr et al., (2005); w_n = natural water content; γ_b = bulk unit weight; γ_d = dry unit weight; LL = liquid limit; PL = plastic limit; PI = plasticity index; LI = liquidity index; S_u (UU) = undrained shear strength (unconsolidated, undrained); S_{up} (CU) = peak undrained shear strength (consolidated, undrained); φ_P = friction angle (peak); φ [DS] = friction angle [Direct Shear]; $\varphi_{C=0}$ [DS] = friction angle @ C_u = 0 [Direct Shear]; C_u = Cohesion; e_0 = initial void ratio; Cc = compression index; Cr = recompression index; $p'_{c\text{-}T}$ = preconsolidation pressure (Tangent); $p'_{c\text{-}C}$ = preconsolidation pressure (Casagrande); $p'_{c\text{-}SE}$ = preconsolidation pressure (Strain Energy)

Table 4.14: Lab Data, Probability Distribution for Warrensburg (Classified)

Parameter / Soil Type	Count	β_1	β_2	Pearson Type	Reported Distribution / Reference
(a) MOISTURE CONTENT AND UNIT WEIGHT					
Natural Water Content, w_n (%)					

Table 4.14 Cont'd: Lab Data, Probability Distribution for Warrensburg (Classified)

Parameter / Soil Type	Count	β_1	β_2	Pearson Type	Reported Distribution / Reference
CL	297	0.04	4.4	IV	Beta/2
CH	8	0.9	3.0	I (J)	Beta/2
Bulk Unit Weight, γ_b(pcf)					
CL	207	0.3	3.3	I(∩)	N/3
CH	5	1.3	3.7	I (J)	N/3
Dry Unit Weight, γ_d (pcf)					
CL	207	0.01	3.2	IV	N/3
(b) ATTERBERG LIMITS					
Liquid Limit, LL (%)					
CL	258	0.1	3.6	IV	N/1,2
CH	9	4.5	7.0	I (J)	N/1,2
Plastic Limit, PL (%)					
CL	257	2.3	7.5	VI	N/1;LN/2
CH	9	0.7	2.4	I (U)	N/1;LN/2
Plasticity Index, PI (%)					
CL	257	0.1	3.4	IV	
CH	9	0.3	1.3	I (U)	
Liquidity Index, LI					
CL	178	0.04	2.04	I(∩)	
(c) STRENGTH PARAMETERS					
Undrained Shear Strength, S_u [UU] (psf)					
CL	51	0.7	2.8	I (J)	N, LN/1
Unconfined Compressive Strength, Q_u/2 (psf)					
CL	16	2.0	6.2	VI	LN/3
Undrained Shear Strength (peak), S_{up} [CU] (psf)					
CL	42	0.6	2.7	I (J)	N, LN/1
Friction Angle (peak), φ_p [CU] (deg)					
CL	42	0.3	2.4	I(∩)	
Friction Angle, φ [DS] (deg)					
CL	19	0.2	2.7	I(∩)	LN/3
Friction Angle (@ cohesion = 0), $\varphi_{C=0}$ [DS] (deg)					
CL	19	0.4	3.2	I(∩)	N/3
Cohesion, C_u [DS] (psf)					
CL	19	0.2	2.1	I (J)	

Table 4.14 Cont'd: Lab Data, Probability Distribution for Warrensburg (Classified)

Parameter / Soil Type	Count	β_1	β_2	Pearson Type	Reported Distribution / Reference
(d) CONSOLIDATION PARAMETERS					
Initial Void Ratio, e_0					
CL	207	0.2	3.5	IV	N/1; LN/2
Compression Index, Cc					
CL	52	40.5	46.8	I (U)	Gamma/2
Recompression Index, Cr					
CL	51	35.1	42.4	I (J)	
Preconsolidation Pressure (Tangent), p'_{c-T} (psf)					
CL	8	0.01	2.1	I(∩)	LN/1
Preconsolidation Pressure (Casagrande), p'_{c-C} (psf)					
CL	51	0.6	3.7	IV	LN/1
Preconsolidation Pressure (Strain Energy), p'_{c-SE} (psf)					
CL	29	0.06	3.0	IV	LN/1
Notes: 1 = Uzielli et al., (2007); 2 = Corotis et al., (1975); 3 = Loehr et al., (2005)					

Table 4.15: Lab Data, Probability Distribution for Warrensburg (Classified, In-Situ State: AGWL)

Parameter / Soil Type	Count	β_1	β_2	Pearson Type	Reported Distribution / Reference
(a) MOISTURE CONTENT AND UNIT WEIGHT					
Natural Water Content, w_n (%)					
CL	52	0.1	4.0	IV	Beta/2
CH	8	0.9	3.0	I (J)	Beta/2
Bulk Unit Weight, γ_b (pcf)					
CL	23	1.6	4.4	I (J)	N/3
CH	5	1.3	3.7	I (J)	N/3
Dry Unit Weight, γ_d (pcf)					
CL	23	0.6	3.7	I(∩)	N/3
(b) ATTERBERG LIMITS					

Table 4.15 Cont'd: Lab Data, Probability Distribution for Warrensburg (Classified, In-Situ State: AGWL)

Parameter / Soil Type	Count	β_1	β_2	Pearson Type	Reported Distribution / Reference
Liquid Limit, LL (%)					
CL	39	0.004	2.7	II	N/1, 2
CH	8	8.0	11.0	I (J)	N/1, 2
Plastic Limit, PL (%)					
CL	39	0.2	3.4	V	N/1; LN/2
CH	8	1.3	3.1	I (J)	N/1; LN/2
Plasticity Index, PI (%)					
CL	39	0.03	2.2	I(∩)	
Liquidity Index, LI					
CL	7	0.89	3.97	I(∩)	
(c) STRENGTH PARAMETERS					
Unconfined Compressive Strength, $Q_u/2$ (psf)					
CL	7	3.0	6.5	I (J)	LN/3
(d) CONSOLIDATION PARAMETERS					
Initial Void Ratio, e_0					
CL	23	1.7	4.6	I (J)	N/1; LN/2
Compression Index, Cc					
CL	5	0.4	2.9	I(∩)	Gamma/2
Recompression Index, Cr					
CL	5	1.1	4.4	I(∩)	
Preconsolidation Pressure (Casagrande), $\sigma'_{p\text{-}C}$ (psf)					
CL	5	3.2	6.6	I (J)	LN/1
Notes: 1 = Uzielli et al., (2007); 2 = Corotis et al., (1975); 3 = Loehr et al., (2005)					

Table 4.16: Lab Data, Probability Distribution for Warrensburg (Classified, In-Situ State: BGWL)

Parameter / Soil Type	Count	β_1	β_2	Pearson Type	Reported Distribution / Reference
(a) MOISTURE CONTENT AND UNIT WEIGHT					
Natural Water Content, w_n (%)					
CL	248	0.1	4.5	IV	Beta/2
Bulk Unit Weight, γ_b(pcf)					

Table 4.16 Cont'd: Lab Data, Probability Distribution for Warrensburg (Classified, In-Situ State: BGWL)

Parameter / Soil Type	Count	β_1	β_2	Pearson Type	Reported Distribution / Reference
CL	185	0.2	3.3	III	N/3
Dry Unit Weight, γ_d (pcf)					
CL	185	0.03	3.3	IV	N/3
(b) ATTERBERG LIMITS					
Liquid Limit, LL (%)					
CL	224	0.3	4.0	IV	N/1, 2
Plastic Limit, PL (%)					
CL	223	2.2	7.0	VI	N/1; LN/2
Plasticity Index, PI (%)					
CL	223	0.2	4.0	IV	
Liquidity Index, LI					
CL	156	0.03	1.99	I(∩)	
(c) STRENGTH PARAMETERS					
Undrained Shear Strength, S_u [UU] (psf)					
CL	51	0.7	2.8	I (J)	N,LN/1
Unconfined Compressive Strength, $Q_u/2$ (psf)					
CL	10	0.3	1.6	I (U)	LN/3
Undrained Shear Strength (peak), S_{up} [CU] (psf)					
CL	41	0.6	2.7	I (J)	N,LN/1
Friction Angle (peak), φ_p [CU] (deg)					
CL	41	0.3	2.4	I(∩)	
Friction Angle, φ [DS] (deg)					
CL	14	0.02	1.9	I(∩)	
Friction Angle (@ C = 0), $\varphi_{C=0}$ [DS] (deg)					
CL	14	0.003	1.7	I (U)	LN/3
Cohesion, C_u [DS] (psf)					
CL	14	0.2	2.6	I(∩)	N/3
(d) CONSOLIDATION PARAMETERS					
Initial Void Ratio, e_0					
CL	185	0.1	3.2	V	N/1; LN/2
Compression Index, Cc					
CL	47	37.8	43.8	I (J)	Gamma/2
Recompression Index, Cr					
CL	48	33.4	40.0	I (J)	

Table 4.16 Cont'd: Lab Data, Probability Distribution for Warrensburg (Classified, In-Situ State: BGWL)

Parameter / Soil Type	Count	β_1	β_2	Pearson Type	Reported Distribution / Reference
Preconsolidation Pressure (Tangent), $p'_{c\text{-}T}$ (psf)					
CL	27	0.0	2.2	II	LN/1
Preconsolidation Pressure (Casagrande), $p'_{c\text{-}C}$ (psf)					
CL	47	0.5	3.8	IV	LN/1
Preconsolidation Pressure (Strain Energy), $p'_{c\text{-}SE}$ (psf)					
CL	28	0.04	3.0	I(∩)	LN/1
Notes: 1 = Uzielli et al., (2007); 2 = Corotis et al., (1975); 3 = Loehr et al., (2005)					

Table 4.17: CPTu Data, Probability Distribution for Warrensburg (Classified)

Soil Property / Soil Type	Count	β_1	β_2	Pearson Type	Reported Distribution / Reference
(a) CLASSIFIED					
Cone Resistance, q_c (tsf)					
1. Sensitive Fine-Grained	6	0.1	1.5	I (U)	
2. Organic Soil	58	0.3	2.6	I (∩)	
3. Clay	629	0.9	3.6	I (∩)	N, LN/1
4. Clay & Silty Clay	44	4.8	9.7	I (J)	
5. Silty Sand & Sandy Silt	36	7.9	11.8	I (J)	
9. Very Dense/Stiff Soil+	22	0.3	2.1	I (J)	
Side Friction, f_s (tsf)					
1. Sensitive Fine-Grained	6	2.4	4.4	I (J)	
2. Organic Soil	58	1.3	4.2	I (J)	
3. Clay	629	0.8	3.2	I (J)	
4. Clay & Silty Clay	44	7.4	11.6	I (J)	
5. Silty Sand & Sandy Silt	36	4.0	6.0	I (U)	
9. Very Dense/Stiff Soil+	22	0.1	1.8	I (U)	
Corrected Cone Resistance, q_t (tsf)					
1. Sensitive Fine-Grained	6	3.5	6.8	I (J)	
2. Organic Soil	58	0.2	2.5	I (∩)	
3. Clay	629	0.7	3.4	I (J)	N, LN/1
4. Clay & Silty Clay	44	5.0	9.8	I (J)	

Table 4.17 Cont'd: CPTu Data, Probability Distribution for Warrensburg (Classified)

Soil Property / Soil Type	Count	β_1	β_2	Pearson Type	Reported Distribution / Reference
5. Silty Sand & Sandy Silt	36	8.1	12.0	I (J)	
9. Very Dense/Stiff Soil$^+$	22	0.3	2.2	I (J)	
Undrained Shear Strength, S_u (psf)					
1. Sensitive Fine-Grained	6	3.1	6.6	I (J)	
2. Organic Soil	33	7.0	10.6	I (J)	
3. Clay	493	2.0	4.2	I (U)	
4. Clay & Silty Clay	37	3.2	8.5	VI	
9. Very Dense/Stiff Soil$^+$	18	0.02	1.8	I (J)	
Normalized Corrected Cone Resistance, q_t/σ'_{vo}					
1. Sensitive Fine-Grained	6	3.7	7.1	I (J)	
2. Organic Soil	33	3.3	7.8	I (∩)	
3. Clay	493	1.6	4.2	I (J)	
4. Clay & Silty Clay	37	5.5	9.8	I (J)	
5. Silty Sand & Sandy Silt	36	4.4	7.4	I (J)	
9. Very Dense/Stiff Soil$^+$	18	0.1	2.0	I (J)	
(b) CLASSIFIED, IN-SITU STATE: AGWL					
Cone Resistance, q_c (tsf)					
2. Organic Soil	14	0.1	1.8	I (U)	N, LN/1
3. Clay	90	1.0	3.0	I (J)	
9. Very Dense/Stiff Soil$^+$	22	0.3	2.1	I (J)	
Side Friction, f_s (tsf)					
2. Organic Soil	14	0.02	1.8	I (J)	
3. Clay	90	1.2	3.2	I (J)	
9. Very Dense/Stiff Soil$^+$	22	0.1	1.8	I (U)	
Corrected Cone Resistance, q_t (tsf)					
2. Organic Soil	14	0.1	1.5	I (∩)	N, LN/1
3. Clay	90	1.0	3.0	I (J)	
9. Very Dense/Stiff Soil$^+$	22	0.3	2.2	I (J)	
Undrained Shear Strength, S_u (psf)					
3. Clay	67	0.5	2.7	I (∩)	
9. Very Dense/Stiff Soil$^+$	18	0.016	1.8	I (J)	

Table 4.17 Cont'd: CPTu Data, Probability Distribution for Warrensburg (Classified)

Soil Property / Soil Type	Count	β_1	β_2	Pearson Type	Reported Distribution / Reference
Normalized Corrected Cone Resistance, q_t/σ'_{vo}					
2. Organic Soil	8	1.7	4.7	I (J)	
3. Clay	67	0.7	2.3	I (U)	
9. Very Dense/Stiff Soil^{+}	18	0.1	2.0	I (J)	
(c) CLASSIFIED, IN-SITU STATE: BGWL					
Cone Resistance, q_c (tsf)					
1. Sensitive Fine-Grained	6	0.1	1.5	I (U)	
2. Organic Soil	44	0.2	2.3	I (∩)	
3. Clay	551	18.2	31.0	VI	N, LN/1
4. Clay & Silty Clay	44	4.8	9.7	I (J)	
5. Silty Sand & Sandy Silt	36	7.9	11.8	I (J)	
Side Friction, f_s (tsf)					
1. Sensitive Fine-Grained	6	2.4	4.4	I (J)	
2. Organic Soil	44	0.8	3.4	I (∩)	
3. Clay	551	0.8	3.1	I (J)	
4. Clay & Silty Clay	44	7.4	11.6	I (J)	
5. Silty Sand & Sandy Silt	36	4.0	6.0	I (U)	
Corrected Cone Resistance, q_t (tsf)					
1. Sensitive Fine-Grained	6	3.5	6.8	I (J)	
2. Organic Soil	44	0.2	2.4	I (∩)	
3. Clay	551	18.8	31.9	VI	N, LN/1
4. Clay & Silty Clay	44	5.0	9.8	I (J)	
5. Silty Sand & Sandy Silt	36	8.1	12.0	I (J)	LN/1
Undrained Shear Strength, S_u (psf)					
1. Sensitive Fine-Grained	6	3.1	6.6	I (J)	
2. Organic Soil	44	7.1	9.8	I (J)	
3. Clay	551	5.8	11.1	I (J)	
4. Clay & Silty Clay	37	3.2	8.5	VI	
Normalized Corrected Cone Resistance, q_t/σ'_{vo}					
1. Sensitive Fine-Grained	6	3.7	7.1	I (J)	
2. Organic Soil	25	2.4	6.1	I (∩)	
3. Clay	438	14.3	22.6	I (J)	
4. Clay & Silty Clay	37	5.5	9.8	I (J)	
5. Silty Sand & Sandy Silt	36	4.4	7.4	I (J)	
Notes: 1 = Uzielli et al., (2007); σ'_{vo} = effective overburden pressure					

4.3.2. Effect of In-Situ State on Probability Distribution. The effect of in-situ state on the probability distribution function/type for parameters was evaluated by determining the probability distribution with both unclassified and classified data. Classification in this case is according to in-situ state – above GWL and below GWL. The investigation was carried out because the soil behavior is known to be affected by in-situ state (Uzielli et al., 2007) – saturated and unsaturated soil behaviors are known to differ. The investigation was carried out with both the research (S&T, and MU) data and all the study data for S_u (UU) and all the study data for all the parameters considered.

The results of the investigation/analyses with the research data for S_u (UU) are presented in Figure 4.2 and Table 4.18. Based on research data, Figure 4.2 presents the location of S_u (UU) on the Pearson space as a result of in-situ state based on data of locations.

Table 4.18: Effect of In-Situ State on Pearson Types for S_u (UU) – Research Data Only

ID	Description	Pearson's Type
W-BGWL-CL	Warrensburg - Below GWL - CL	I(J): J-Shaped
SC-BGWL-CH	St. Charles - Below GWL - CH	III
P-BGWL-CH	Pemiscot - Below GWL - CH	II
W-CL	Warrensburg - CL	I(J): J-Shaped
SC-CH	St. Charles - CH	III
P-CH	Pemiscot - CH	II

The results of the investigation/analyses with the data for Warrensburg are presented in Table 4.10 and Figure 4.1, and Tables 4.11 to 4.17. The results of the investigation/analyses with the data from all the locations are presented in Appendices C and D. An excerpt from Tables 4.11 to 4.17 showing the effect of in-situ state on the probability distribution of some randomly chosen parameters is presented in Table 4.19.

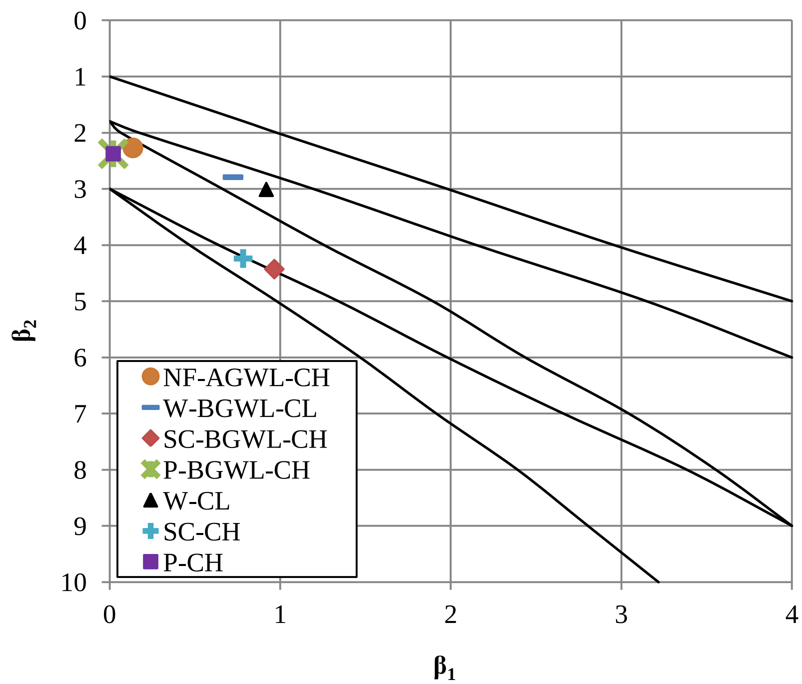


Figure 4.2: Pearson's Distribution Space for S_u (UU) – Locations

Table 4.19: Effect of In-Situ State on Pearson Types for Some Parameters – Warrensburg

Parameter / Classification	NONE		AGWL		BGWL	
	Count	Pearson Type	Count	Pearson Type	Count	Pearson Type
γ_b (pcf)	231	IV	30	IV	185	III
LL (%)	291	VI	53	I(∩)	224	IV
Su [UU] (psf)	68	I(J)	*	*	51	I(J)
$p'_{c\text{-}C}$ (psf)	52	IV	57	I (J)	47	IV
w_n - CL	297	IV	52	IV	248	IV
PL - CL	257	I(∩)	39	V	223	VI

Table 4.19 Cont'd: Effect of In-Situ State on Pearson Types for some Parameters – Warrensburg

Parameter / Classification	NONE		AGWL		BGWL	
	Count	Pearson Type	Count	Pearson Type	Count	Pearson Type
q_t - Clay	629	I(J)	90	I(J)	551	I(J)
φ_p [CU] - CL	42	I(∩)	*	*	41	I(∩)
e_0	231	IV	30	I(∩)	185	V
q_c - Clay	629	I(J)	90	I(J)	551	VI
f_s - Clay	629	I(J)	90	I(J)	551	VI

Note: γ_b = bulk unit weight; LL = liquid limit; PL = plastic limit; S_u (UU) = undrained shear strength (unconsolidated, undrained); φ_P = friction angle (peak); e_0 = initial void ratio; $p'_{c\text{-}C}$ = preconsolidation pressure (Casagrande)

4.4. CORRELATION OF SOIL PROPERTIES

4.4.1. General. In geotechnical engineering practice, the use of correlations and empirical relationships may provide a fast, cost-effective means of predicting parameters provided the appropriate correlations are employed. They are particularly useful in preliminary studies, or when, due to time and/or financial constraints, a thorough geotechnical investigation cannot to be conducted. In this section, investigations are led to develop correlation matrices showing the correlation between parameters, developing correlations between parameters, and examine the validity of published empirical relationships.

4.4.2. Correlation Matrix. An analysis, based on the research (S&T and MU) data of TRGP, was led to develop a correlation matrix showing the correlation between parameters. The parameters investigated are Atterberg limits (PL, LL, PI, LI), natural moisture content, w_n, initial void ratio, e_0, unit weight (bulk, γ_b, and dry, γ_d), undrained shear strength (unconsolidated, undrained), S_u (UU), undrained shear strength (consolidated, undrained), S_{up} (CU), compression index, Cc, recompression index, Cr, peak friction angle [from single S_u (CU) test], ϕ_p, preconsolidation pressure (Casagrande), p'_{c-C}.

The correlation matrix analysis for this study was carried out using the Microsoft Excel 2007 add-in for data analysis. Knowledge of the degree of correlation between parameters is required in higher level RBD implementations.

The correlation matrix showing the correlation between parameters based on the Research data of TRGP is presented in Tables 4.20 to 4.23 for Warrensburg, St. Charles, New Florence, and Pemiscot, respectively. The data count for Warrensburg, St. Charles, New Florence, and Pemiscot was 48, 41, 38, and 89, respectively. The comprehensive results including the results for various locations are presented in Appendix H.

Table 4.20: Correlation Matrix for Warrensburg (Data Count = 48)

	LL	PL	PI	LI	w_n	e_0	γ_b	γ_d	S_u (UU)	S_{up} (CU)	φ_p	Cc	Cr	p'_{c-C}
LL	1.00													
PL	0.73	1.00												
PI	0.32	-0.42	1.00											
LI	-0.80	-0.63	-0.18	1.00										
w_n	-0.16	-0.01	-0.20	0.65	1.00									
e_0	-0.09	0.04	-0.17	0.34	0.60	1.00								
γ_b	0.06	0.09	-0.04	-0.18	-0.31	-0.91	1.00							
γ_d	0.07	0.05	0.02	-0.34	-0.59	-0.98	0.94	1.00						
S_u (UU)	0.32	0.12	0.25	-0.43	-0.50	-0.55	0.45	0.53	1.00					
S_{up} (CU)	-0.10	-0.31	0.29	0.11	0.05	0.32	-0.47	-0.40	-0.28	1.00				
φ'_p	-0.20	-0.15	-0.06	0.34	0.40	0.46	-0.41	-0.46	-0.47	0.48	1.00			
Cc	0.08	0.02	0.08	-0.04	0.04	-0.07	0.07	0.04	0.43	-0.05	-0.27	1.00		
Cr	0.09	-0.07	0.21	0.0002	0.13	0.18	-0.18	-0.20	-0.19	0.37	0.20	-0.16	1.00	
p'_{c-C}	0.34	0.13	0.26	-0.35	-0.30	-0.001	-0.19	-0.05	0.16	0.18	-0.10	-0.23	0.26	1.00

Table 4.21: Correlation Matrix for St. Charles (Data Count = 41)

	LL	PL	PI	LI	w_n	e_0	γ_b	γ_d	S_u (UU)	S_{up} (CU)	φ_p	Cc	Cr	$p'_{c\text{-}C}$
LL	1.00													
PL	0.90	1.00												
PI	0.97	0.76	1.00											
LI	-0.45	-0.45	-0.41	1.00										
w_n	0.78	0.83	0.69	0.04	1.00									
e_0	0.83	0.82	0.76	-0.03	0.94	1.00								
γ_b	-0.85	-0.78	-0.81	0.23	-0.77	-0.92	1.00							
γ_d	-0.85	-0.84	-0.78	0.09	-0.93	-0.99	0.95	1.00						
S_u (UU)	-0.62	-0.62	-0.56	0.10	-0.59	-0.64	0.63	0.65	1.00					
S_{up} (CU)	-0.37	-0.42	-0.30	0.07	-0.23	-0.34	0.42	0.36	0.35	1.00				
φ'_p	-0.20	-0.19	-0.19	-0.22	-0.25	-0.32	0.25	0.28	0.32	0.14	1.00			
Cc	-0.34	-0.15	-0.43	0.58	0.07	-0.02	0.22	0.08	-0.09	0.04	-0.03	1.00		
Cr	0.05	0.37	-0.13	0.04	0.37	0.36	-0.27	-0.35	-0.21	-0.20	0.07	0.30	1.00	
$p'_{c\text{-}C}$	-0.58	-0.59	-0.52	0.27	-0.40	-0.50	0.61	0.52	0.57	0.51	0.15	0.21	-0.23	1.00

Table 4.22: Correlation Matrix for New Florence (Data Count = 38)

	LL	PL	PI	LI	w_n	e_0	γ_b	γ_d	S_u (UU)	S_{up} (CU)	φ_p	Cc	Cr	$p'_{c\text{-}C}$
LL	1.00													
PL	0.68	1.00												
PI	0.98	0.50	1.00											
LI	-0.45	-0.24	-0.45	1.00										
w_n	0.68	0.68	0.60	0.28	1.00									
e_0	0.54	0.58	0.47	0.08	0.73	1.00								
γ_b	-0.39	-0.40	-0.35	0.10	-0.42	-0.88	1.00							
γ_d	-0.54	-0.55	-0.47	-0.02	-0.69	-0.99	0.91	1.00						
S_u (UU)	-0.39	-0.45	-0.33	-0.10	-0.61	-0.47	0.22	0.44	1.00					
S_{up} (CU)	-0.35	-0.34	-0.31	-0.01	-0.37	-0.001	-0.11	-0.01	0.07	1.00				
φ'_p	-0.64	-0.30	-0.66	0.16	-0.46	-0.17	0.06	0.17	0.08	0.53	1.00			
Cc	-0.08	-0.24	-0.03	-0.21	-0.33	0.01	-0.16	-0.03	0.14	0.78	0.34	1.00		
Cr	0.38	0.05	0.43	-0.16	0.16	-0.09	0.05	0.06	0.16	-0.73	-0.49	-0.45	1.00	
$p'_{c\text{-}C}$	-0.24	-0.42	-0.16	-0.09	-0.37	-0.01	-0.09	-0.01	0.13	0.92	0.27	0.74	-0.59	1.00

Table 4.23: Correlation Matrix for Pemiscot (Data Count = 89)

	LL	PL	PI	LI	w_n	e_0	γ_b	γ_d	S_u (UU)	S_{up} (CU)	φ_p	Cc	Cr	$p'_{c\text{-}C}$
LL	1.00													
PL	0.49	1.00												
PI	0.96	0.23	1.00											
LI	-0.29	-0.32	-0.22	1.00										
w_n	0.84	0.39	0.82	0.11	1.00									
e_0	0.86	0.34	0.85	0.04	0.96	1.00								
γ_b	-0.79	-0.21	-0.81	0.17	-0.75	-0.88	1.00							
γ_d	-0.77	-0.38	-0.74	-0.09	-0.87	-0.92	0.78	1.00						
S_u (UU)	-0.14	0.30	-0.25	-0.05	-0.17	-0.26	0.35	0.20	1.00					
S_{up} (CU)	-0.42	-0.33	-0.36	0.21	-0.30	-0.33	0.35	0.35	-0.13	1.00				
φ'_p	-0.20	-0.30	-0.12	-0.12	-0.37	-0.36	0.29	0.40	-0.12	0.15	1.00			
Cc	0.77	0.40	0.74	-0.13	0.76	0.74	-0.62	-0.69	-0.21	-0.28	-0.26	1.00		
Cr	0.51	0.40	0.45	-0.40	0.35	0.37	-0.40	-0.33	-0.14	-0.18	-0.01	0.44	1.00	
$p'_{c\text{-}C}$	0.36	0.29	0.31	-0.14	0.32	0.29	-0.17	-0.35	-0.01	-0.19	-0.33	0.59	0.09	1.00

4.4.3. Development of Correlations Between Parameters. The use of simply-obtained parameters, like Atterberg limits, to predict the value of complexly-obtained parameters, like undrained shear strength, S_u, if applied under appropriate conditions is acceptable in geotechnical engineering practice if applied under appropriate conditions. An analysis, based on the research (S&T and MU) data of TRGP, was led to develop correlation models between Atterberg limits (Plasticity Index, PI) the normalized (with respect to effective overburden pressure) undrained shear strength, (S_u/σ'). This analysis was carried out in two steps: first, with unclassified (with respect to soil classification type) data; and then with classified data. This analysis was carried out using Microsoft Excel 2007 in which curves are fitted using the method of Ordinary Least Square (OLS).

The results of the investigation into the development of correlation models for Atterberg limits (Plasticity Index, PI) and normalized (with respect to effective overburden pressure) undrained shear strength, (S_u/σ') are presented in Figures 4.3 to 4.6, Figures 4.7 to 4.9, and Table 4.24.

The correlation models developed for the PI and (S_u/σ') irrespective of soil classification type is presented in Figures 4.3 to 4.6 for the Warrensburg, St. Charles, New Florence and Pemiscot, respectively. The correlation models developed for the PI and (S_u/σ') with respect to soil classification type is presented in Figures 4.7 to 4.9 for the CL, CH, and MH, respectively. The correlation models developed for the PI and (S_u/σ') with respect to soil classification types including their respective coefficient of determination are presented in Table 4.24.

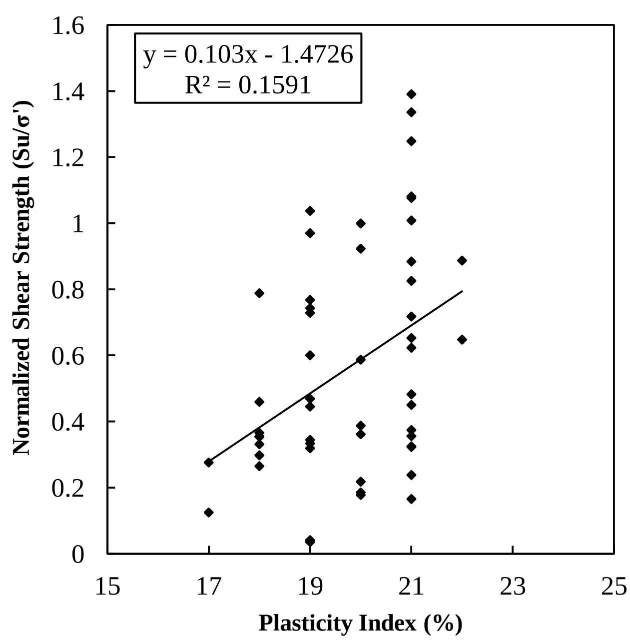


Figure 4.3: Correlations Model for Unclassified (in Terms of Both In-Situ State and Soil Classification Type) PI and (S_u/σ') – Warrensburg

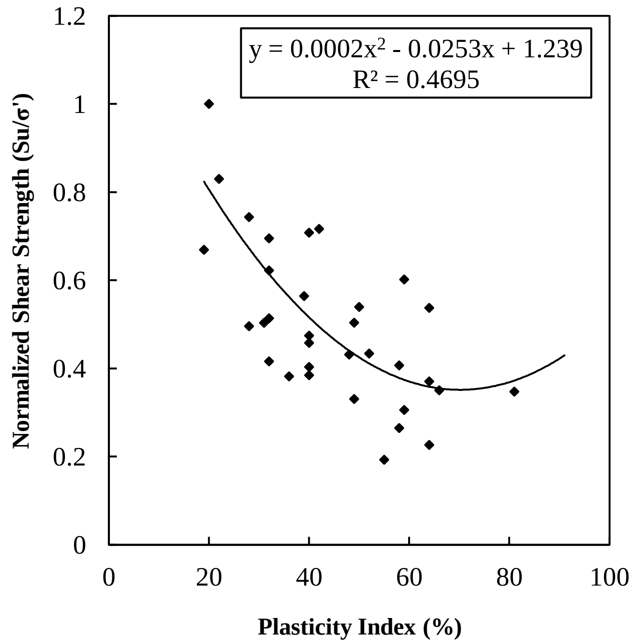


Figure 4.4: Correlations Model for Unclassified (in Terms of Both In-Situ State and Soil Classification Type) PI and (S_u/σ') - St. Charles

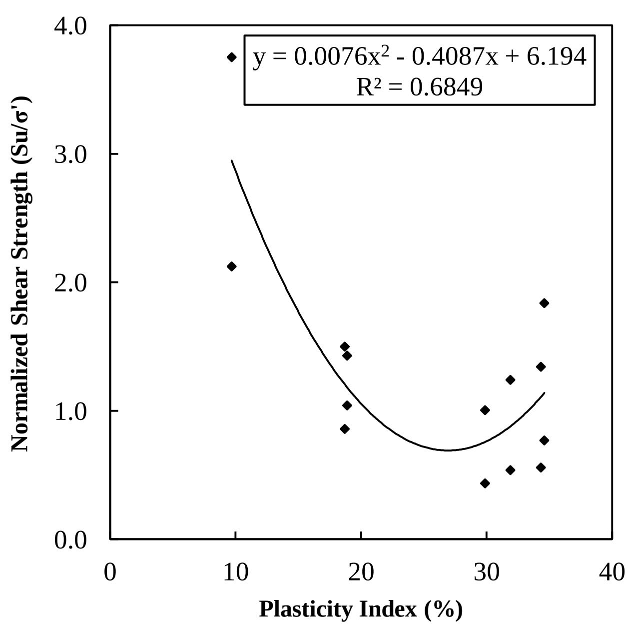


Figure 4.5: Correlations Model for Unclassified (in Terms of Both In-Situ State and Soil Classification Type) PI and (S_u/σ') - New Florence

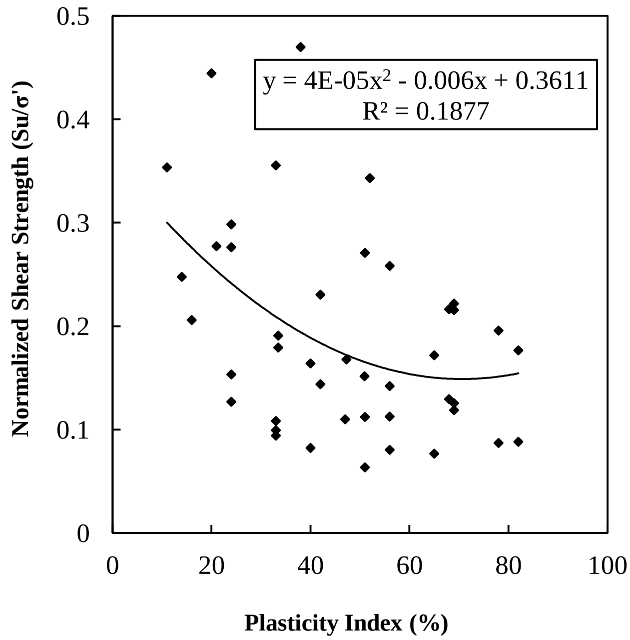


Figure 4.6: Correlations Model for Unclassified (in Terms of Both In-Situ State and Soil Classification Type) PI and (S_u/σ') – Pemiscot

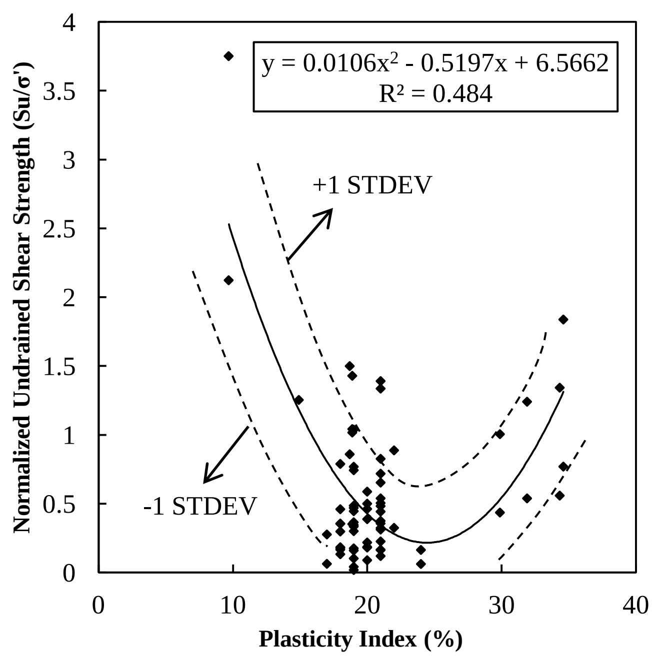


Figure 4.7: Correlations Model for Classified (in Terms of Soil Classification Type Only) PI and (S_u/σ') – CL

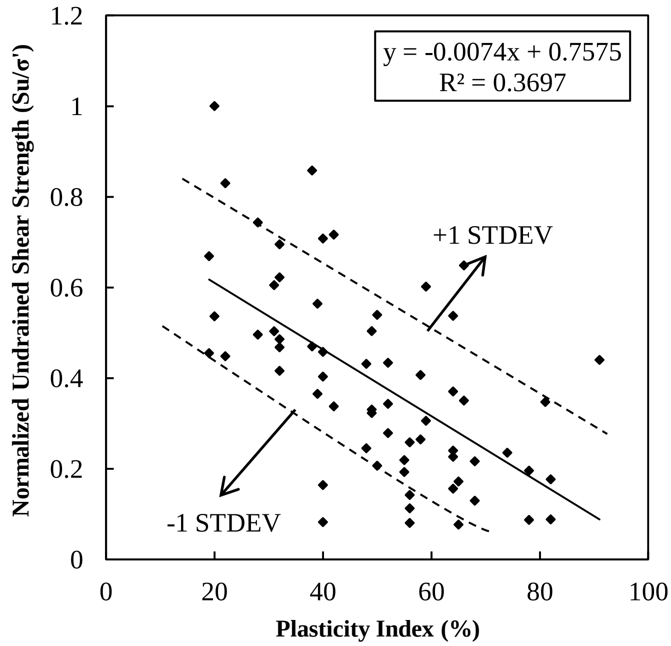


Figure 4.8: Correlations Model for Classified (in Terms of Soil Classification Type Only) PI and (S_u/σ') – CH

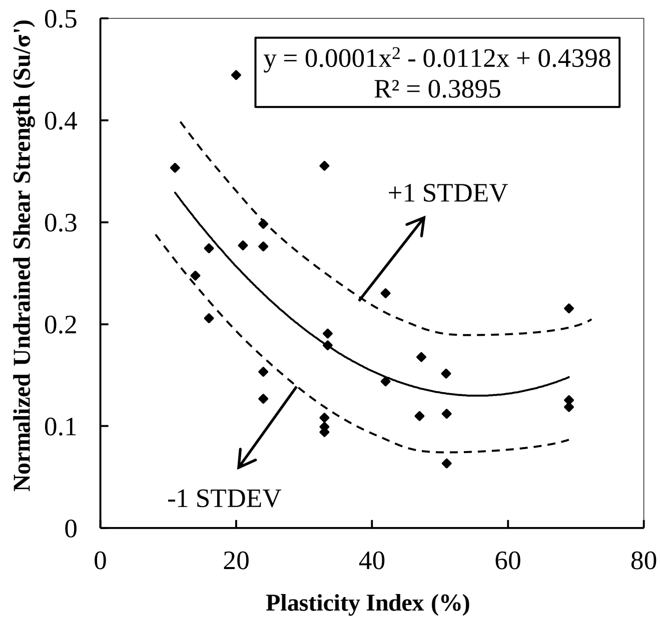


Figure 4.9: Correlations Model for Classified (in Terms of Soil Classification Type Only) PI and (S_u/σ') - MH

Table 4.24: Correlation Models for the PI and (S_u/σ') Based on Soil Classification Type

Soil Type	Correlation Equation	Correlation Coefficient, R^2
CL	$S_u/\sigma' = 0.0106\ PI^2 - 0.5197\ PI + 6.5662$	0.48
CH	$S_u/\sigma' = -0.0074\ PI + 0.7575$	0.37
MH	$S_u/\sigma' = 0.0001\ PI^2 - 0.0112\ PI + 0.4398$	0.39

4.4.4. Assessment of the Validity of Published Empirical Relationships Between Parameters. Analyses, based on all data – TRGP and MoDOT historical data, were led to assess the validity of published empirical relationships between parameters. To simplify the investigation, published empirical relationships were grouped into two: those relating strength parameters to simply obtained parameters; and those relating consolidation parameters to simply obtained parameters. The empirical relationships relating strength parameters to simply obtained parameters were investigated in terms of both S_u (UU) and S_u (CU). The empirical relationships assessed for validity are presented in Tables 4.25 and 4.26 for the strength and consolidation parameters, respectively. Empirical relationship #10 in Table 4.25 was assessed for S_u (CU) only. For the purposes of these analyses, the empirical relationships were reconstituted to yield a straight line when plotted.

For correlations with strength parameters, the validity of nine (Equations 1 to 9, Table 4.25) empirical relationships was assessed for S_u (UU), while the validity of ten (see Table 4.25) empirical relationships was assessed for S_u (CU). For the correlations with consolidation parameters (see Table 4.26), the validity of six empirical relationships was assessed for Cc while the validity of five empirical relationships was assessed for Cr.

Table 4.25: Strength Parameters – Published Empirical Correlations Evaluated

#	Equation	Reference	Remarks
1	$S_u/p'_o = 0.45(I_P)^{1/2}$	Bjerrum and Simons (1960)	NC, $I_P > 0.5$, (decimal)
2	$S_u/p'_o = 0.18(I_L)^{1/2}$	Bjerrum and Simons (1960)	NC, $I_L > 0.5$, (decimal)
3	$S_u/p'_o = 0.11 + 0.0037I_P$	Skempton and Henkel (1953)	NC, I_P (percent)
4	$S_u/p'_o = 0.5w_L$	Karlsson and Viberg (1967)	NC, $w_L > 0.2$, (decimal)
5	$(S_u/\sigma'_z)_{nc} = 0.11 + 0.0037PI$	Skempton (1957)	NC
6	$S_u/p'_o = 0.129 + 0.00435PI$	Worth and Houlsby (1985)	NC
7	$S_u/\sigma'_z = (0.23 \pm 0.04)OCR^{0.8}$	Jamiolkowski et al., (1985)	OC
8	$S_u/\sigma'_{zc} = 0.11 + 0.0037PI$	Skempton (1957)	OC
9	$Su/\sigma'_{zc} = 0.22$	Mesri (1975)	All clays
10	φ' versus PI	Bowles (1997)	Plot

Notes: NC, nc = normally consolidated clay; OC = overconsolidated clay; OCR = overconsolidation ratio; σ'_{zc} = preconsolidation pressure; σ'_z, p'_o = effective overburden pressure; I_P, PI = plasticity index; I_L = liquidity index; w_L = liquid limit; n = data count

Table 4.26: Consolidation Parameters – Published Empirical Correlations Evaluated

#	Equation	Reference	Remarks
1	$Cc = 0.009(LL - 10)$	Skempton (1944)	Remolded clays
2	$Cc = 0.40(e_o - 0.25)$	Azzouz et al., (1976)	
3	$Cc = 0.01(w_n - 5)$	Azzouz et al., (1976)	All clays
4	$Cc = 0.37(e_o + 0.003LL – 0.34)$	Azzouz et al., (1976)	
5	$Cc = 0.5Gs(PI/100) \approx PI/74$	Wood and Worth (1978)	
6	$Cc = 0.00234\ LLGs$	Nagaraj and Murthy (1986)	All clays
7	$Cr = 0.15(e_o + 0.25)$	Azzouz et al., (1976)	All clays
8	$Cr = 0.003(w_n + 10)$	Azzouz et al., (1976)	All clays
9	$Cr = 0.126(e_o + 0.003LL – 0.06)$	Azzouz et al., (1976)	All clays
10	$Cr = 0.00463\ LLGs$	Nagaraj and Murthy (1985)	All clays
11	Cr = Cc 1/5*Cc to 1/10*Cc	Budhu (2008)	

Notes: e_0 = initial void ratio; w_n = natural water content; PI = plasticity index; LL = liquid limit; Gs = specific gravity of solids; Cc = compression index; Cr = recompression index

The results of the investigation led to assess the validity of published empirical relationships between simply-obtained parameters and strength and consolidation parameters, as they relate to the data used in this study, are presented in Tables 4.27 to 4.29 and Figures 4.10 to 4.35. For easy comparison, in Tables 4.27 to 4.29 each empirical relationship being validated is stated while the relationship developed in this study is stated below it. In Figures 4.10 to 4.35 the plots of empirical relationship being validated and the plot developed in this study are shown side by side for easy comparison also.

The results of the validation of empirical relationships relating strength and consolidation parameters to simply obtained parameters are presented in Table 4.27 and Figures 4.10 to 4.16 for S_u (UU), Table 4.28 and Figures 4.17 to 4.24 for S_u (CU), and Table 4.29 and Figures 4.25 to 4.35 for Cc and Cr.

Figures 4.10 to 4.15 presents the relationship between normalized (with respect to effective overburden pressure) undrained shear strength (UU) S_u/σ'_z and PI by Bjerrum and Simmons (1960), S_u/σ'_z and LI by Bjerrum and Simmons (1960), S_u/σ'_z and PI by Skempton and Henkel (1953), Skempton (1957), Worth and Houlsby (1985), S_u/σ'_z and LL by Karlsson and Viberg (1967), and S_u/σ'_z and OCR by Jamiolkowski et al., (1985), respectively. Figures 4.15 and 4.16 presents the relationship between normalized (with respect to preconsolidation pressure) undrained shear strength (UU) S_u/σ'_{zc} and PI by Skempton (1957), and undrained shear strength (UU) S_u and preconsolidation pressure by Mesri (1975), respectively. The foregoing applies to the Figures 4.17 to 4.23 presented for the relationships between undrained shear strength (CU) and other simply-obtained parameters. The difference in this case is the extra relationship between φ and PI by Bowles (1998) presented in Figure 4.24. Figures 4.25 to 4.30 presents the relationship between Cc and LL by Skempton (1944), Cc and e_0 by Azzouz et al., (1976), Cc and w_n by Azzouz et al., (1976), Cc and (e_0 + LL) by Azzouz et al., (1976), Cc and Gs*PI by Wood and Worth (1978) and Cc and LL*GS by Nagaraj and Murthy (1986), respectively. Figures 4.31 to 4.35 presents the relationship between Cr and e_0 by Azzouz et al., (1976), Cr and w_n by Azzouz et al., (1976), Cc and (e_0 + LL) by Azzouz et al., (1976), Cr and LL*GS by Nagaraj and Murthy (1986), and Cr and Cc by Budhu (2008), respectively.

Table 4.27: Strength Parameters – S_u (UU); Published Empirical Correlations and Correlations Developed in this Study

#	Equation	Reference	Remarks
1	$S_u/p'_o = 0.45(I_P)^{1/2}$ $S_u/p'_o = -1.9385(I_P)^{1/2} + 2.286$	Bjerrum and Simons (1960) Present Study	NC, $I_P > 0.5$, (decimal) $R^2 = 0.0879$; n = 60
2	$S_u/p'_o = 0.18(I_L)^{1/2}$ $S_u/p'_o = -0.5118(I_L)^{1/2} + 0.9838$	Bjerrum and Simons (1960) Present Study	NC, $I_L > 0.5$, (decimal) $R^2 = 0.0277$; n = 56
3	$S_u/p'_o = 0.11 + 0.0037I_P$ $S_u/p'_o = = 0.9744 - 0.0022PI$	Skempton and Henkel (1953) Present Study	NC, I_P (percent) $R^2 = 0.0058$; n = 16
4	$S_u/p'_o = 0.5w_L$ $S_u/p'_o = -0.373w_L + 1.1281$	Karlsson and Viberg (1967) Present Study	NC, $w_L > 0.2$, (decimal) $R^2 = 0.0375$; n = 193
5	$(S_u/\sigma'_z)_{nc} = 0.11 + 0.0037PI$ $(S_u/\sigma'_z)_{nc} = 0.9744 - 0.0022PI$	Skempton (1957) Present Study	NC $R^2 = 0.0058$; n = 16
6	$S_u/p'_o = 0.129 + 0.00435PI$ $S_u/p'_o = 0.9744 - 0.0022PI$	Worth and Houlsby (1985) Present Study	NC $R^2 = 0.0058$; n = 16
7	$S_u/\sigma'_z = (0.23 \pm 0.04)OCR^{0.8}$ $S_u/\sigma'_z = (0.0707)OCR^{0.8} + 0.785$	Jamiolkowski et al., (1985) Present Study	OC $R^2 = 0.1613$; n = 132
8	$S_u/\sigma'_{zc} = 0.11 + 0.0037PI$ $S_u/\sigma'_{zc} = 0.953 - 0.0078PI$	Skempton (1957) Present Study	OC $R^2 = 0.1363$; n = 154
9	$S_u = 0.22*\sigma'_{zc}$ $S_u = 0.1363*\sigma'_{zc} + 944.97$	Mesri (1975) Present Study	All clays $R^2 = 0.0405$; n = 212
Notes: NC, nc = normally consolidated clay; OC = overconsolidated clay; OCR = overconsolidation ratio; σ'_{zc} = preconsolidation pressure; σ'_z, p'_o = effective overburden pressure; I_P, PI = plasticity index; I_L = liquidity index; w_L = liquid limit; n = data count			

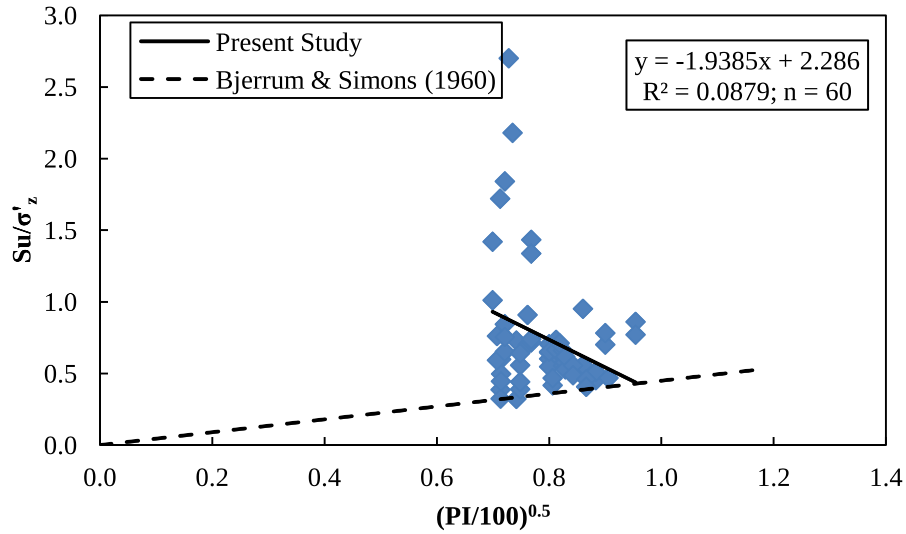


Figure 4.10: S_U (UU), Correlation Between S_u/σ'_z and $(PI/100)^{0.5}$ - Bjerrum and Simons (1960) - #1

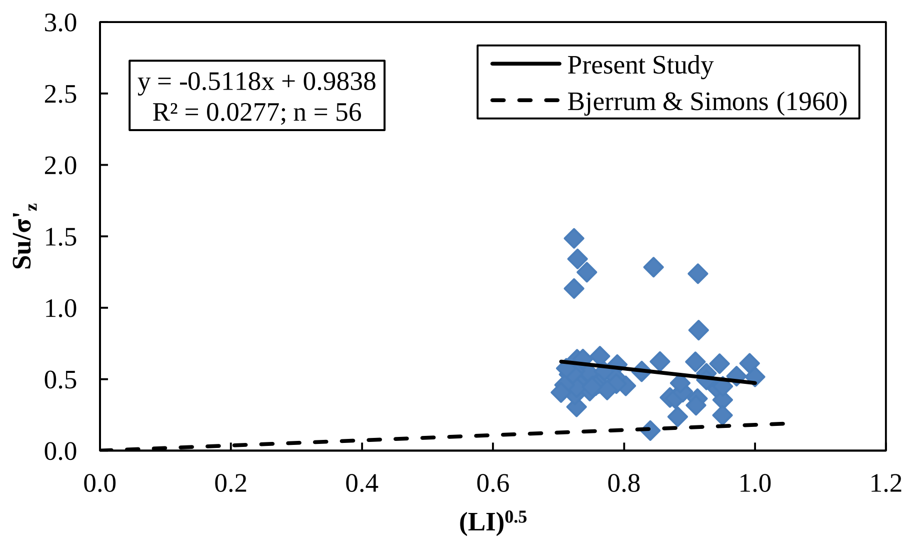


Figure 4.11: S_U (UU), Correlation Between S_u/σ'_z and $(LI)^{0.5}$ - Bjerrum and Simons (1960) - #2

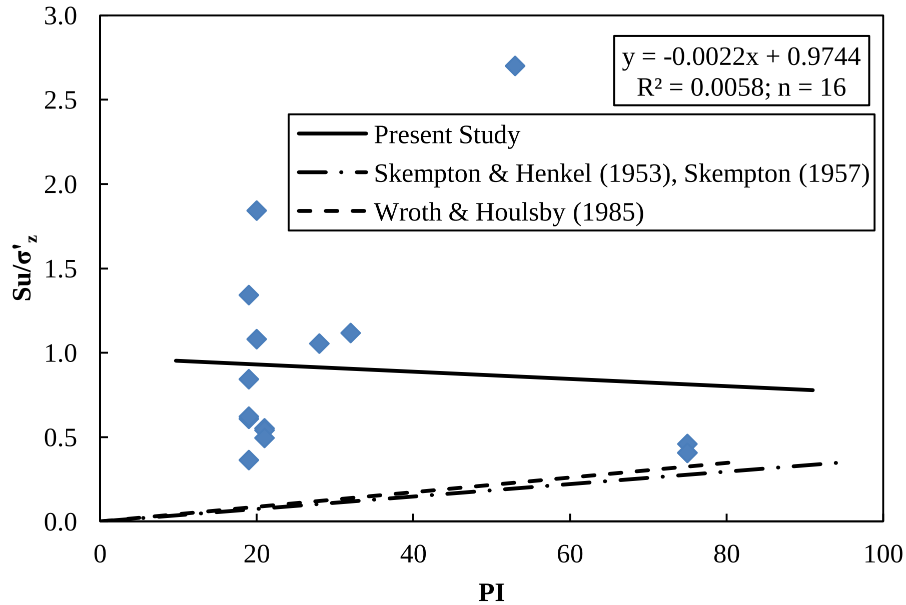


Figure 4.12: S_U (UU), Correlation Between S_u/σ'_z and PI - Skempton and Henkel (1953), Skempton (1957), Worth and Houlsby (1985) - #3, #5, #6

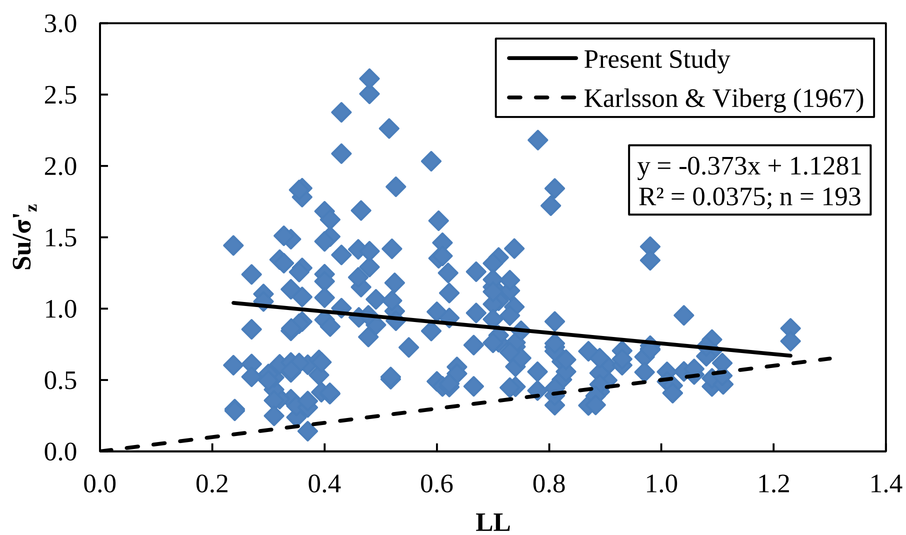


Figure 4.13: S_U (UU), Correlation Between S_u/σ'_z and LL - Karlsson and Viberg (1967) - #4

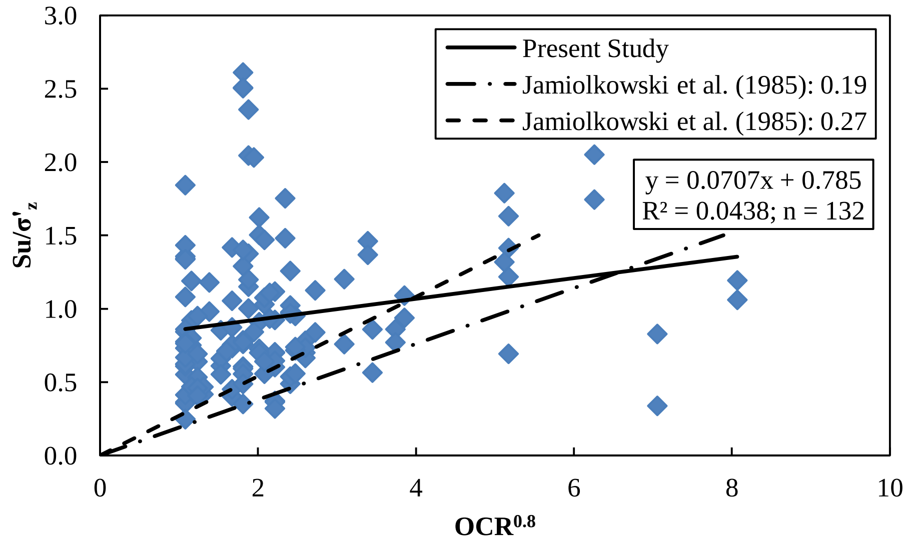


Figure 4.14: S_U (UU), Correlation Between S_u/σ'_z and $OCR^{0.8}$ - Jamiolkowski et al., (1985) - #7

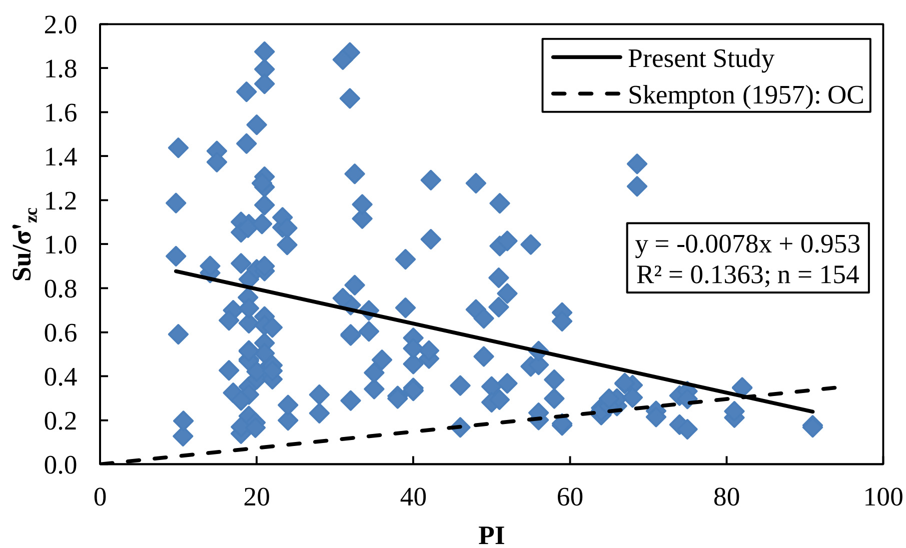


Figure 4.15: S_U (UU), Correlation Between S_u/σ'_{zc} and PI - Skempton (1957) - #8

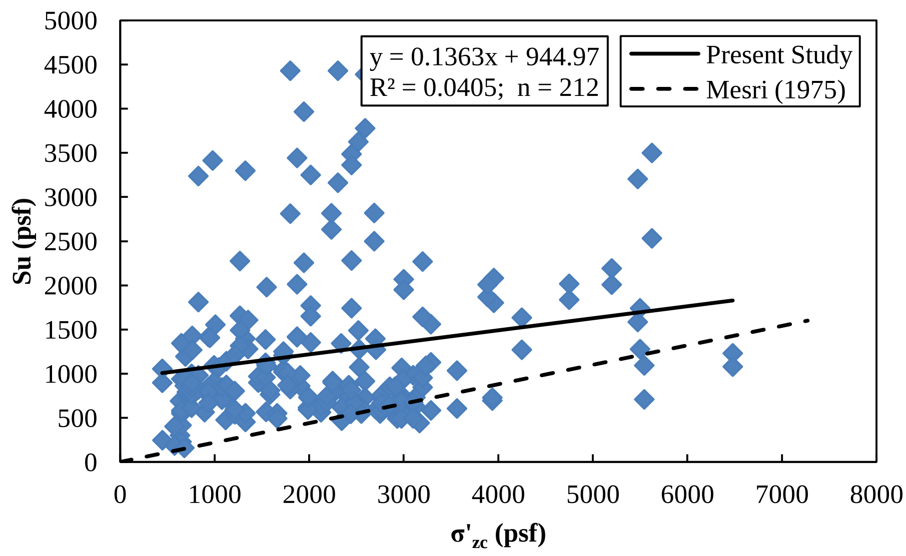


Figure 4.16: S_U (UU), Correlation Between S_u and σ'_{zc} – Mesri (1975) - #9

Table 4.28: Strength Parameters – S_u (CU); Published Empirical Correlations and Correlations Developed in this Study

#	Equation	Reference	Remarks
1	$S_u/p'_o = 0.45(I_P)^{1/2}$ $S_u/p'_o = -3.135(I_P)^{1/2} + 3.8627$	Bjerrum and Simons (1960) Present Study	NC, $I_P > 0.5$, (decimal) $R^2 = 0.06595$; n = 13
2	$S_u/p'_o = 0.18(I_L)^{1/2}$ $S_u/p'_o = -2.3664(I_L)^{1/2} + 3.6434$	Bjerrum and Simons (1960) Present Study	NC, $I_L > 0.5$, (decimal) $R^2 = 0.035$; n = 45
3	$S_u/p'_o = 0.11 + 0.0037I_P$ $S_u/p'_o = 1.5415 + 0.0003I_P$	Skempton and Henkel (1953) Present Study	NC, I_P (percent) $R^2 = 3E\text{-}05$; n = 65
4	$S_u/p'_o = 0.5w_L$ $S_u/p'_o = -2.6186w_L + 3.6434$	Karlsson and Viberg (1967) Present Study	NC, $w_L > 0.2$, (decimal) $R^2 = 0.0311$; n = 78
5	$(S_u/\sigma'_z)_{nc} = 0.11 + 0.0037PI$ $(S_u/\sigma'_z)_{nc} = = 1.5415 + 0.0003I_P$	Skempton (1957) Present Study	NC $R^2 = 3E\text{-}05$; n = 65
6	$S_u/p'_o = 0.129 + 0.00435PI$ $S_u/p'_o = = 1.5415 + 0.0003I_P$	Worth and Houlsby (1985) Present Study	NC $R^2 = 3E\text{-}05$; n = 65

Table 4.28 Cont'd: Strength Parameters – S_u (CU) - Published Empirical Correlations and Correlations Developed in this Study

#	Equation	Reference	Remarks
7	$S_u/\sigma'_z = (0.23 \pm 0.04)OCR^{0.8}$ $S_u/\sigma'_z = (0.1259)OCR^{0.8} + 1.0599$	Jamiolkowski et al., (1985) Present Study	OC $R^2 = 0.0325$; n = 101
8	$S_u/\sigma'_{zc} = 0.11 + 0.0037PI$ $S_u/\sigma'_{zc} = 0.4004 - 0.0038PI$	Skempton (1957) Present Study	OC $R^2 = 0.2654$; n = 88
9	$S_u = 0.22*\sigma'_{zc}$ $S_u = 0.1132*\sigma'_{zc} + 1426.8$	Mesri (1975) Present Study	All clays $R^2 = 0.0999$; n = 171
10	φ' versus PI	Bowles (1997)	Plot
Notes: NC, nc = normally consolidated clay; OC = overconsolidated clay; OCR = overconsolidation ratio; σ'_{zc} = preconsolidation pressure; σ'_z, p'_o = effective overburden pressure; I_P, PI = plasticity index; I_L = liquidity index; w_L = liquid limit; n = data count			

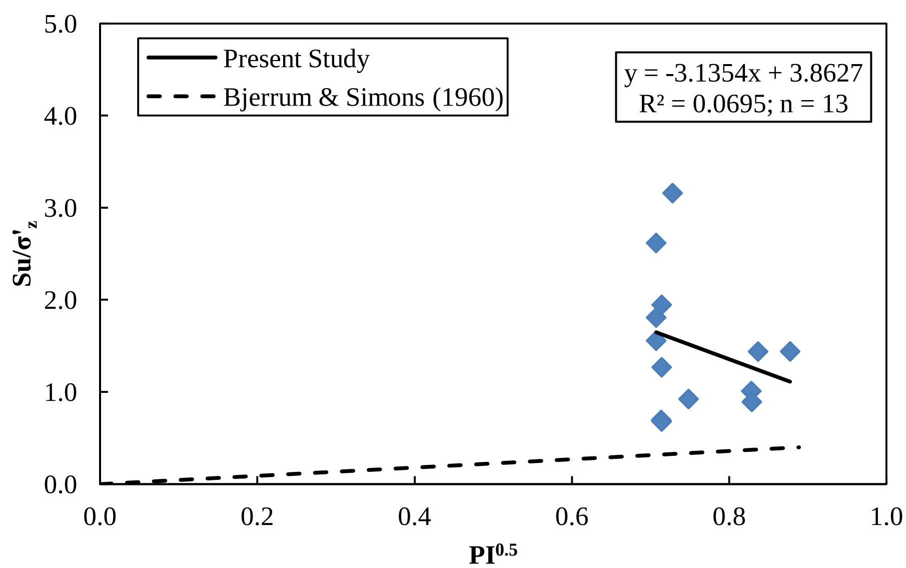


Figure 4.17: S_u (CU), Correlation Between S_u/σ'_z and $(PI/100)^{0.5}$ - Bjerrum and Simons (1960) - #1

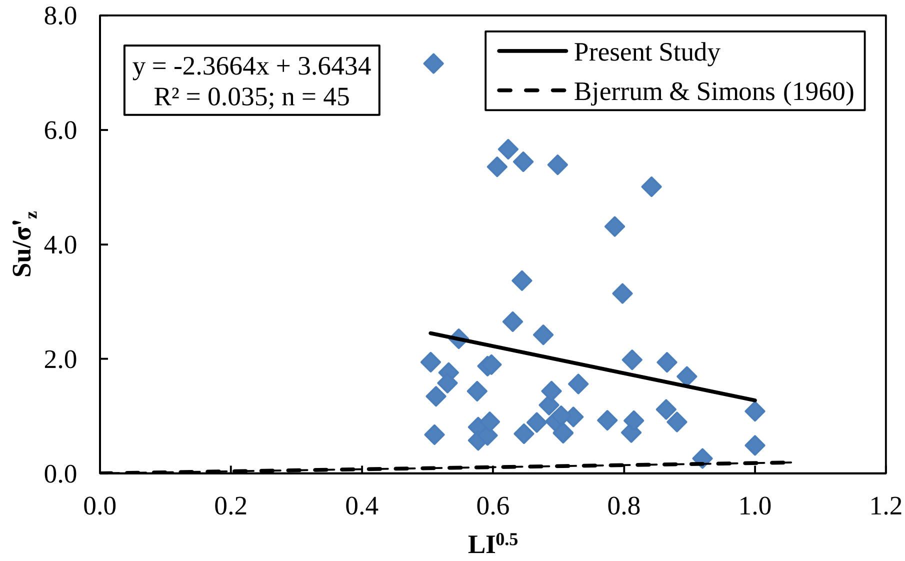


Figure 4.18: S_u (CU), Correlation Between S_u/σ'_z and $(LI)^{1/2}$ - Bjerrum and Simons (1960) - #2

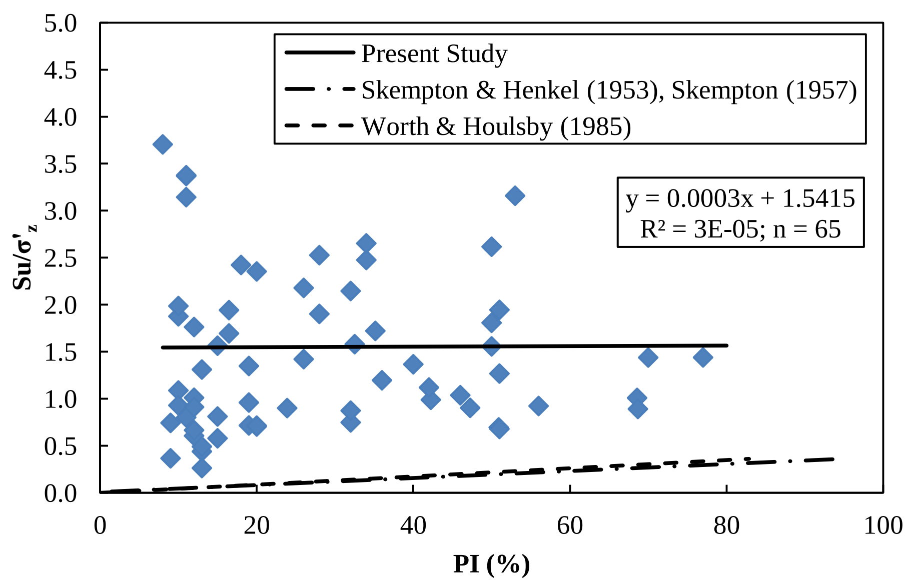


Figure 4.19: S_u (CU), Correlation Between S_u/σ'_z and PI - Skempton and Henkel (1953), Skempton (1957), Worth and Houlsby (1985) - #3, #5, #6

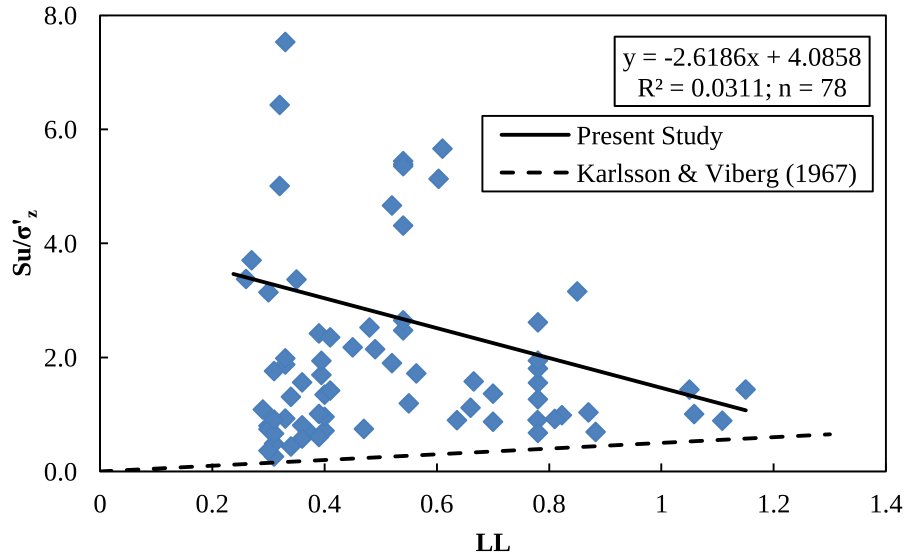


Figure 4.20: S_u (CU), Correlation Between S_u/σ'_z and LL - Karlsson and Viberg (1967) - #4

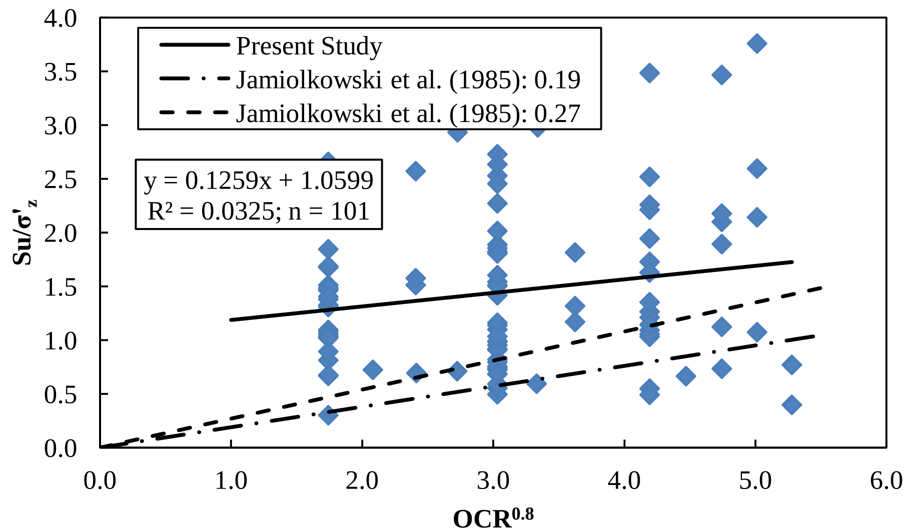


Figure 4.21: S_u (CU), Correlation Between S_u/σ'_z and $OCR^{0.8}$ - Jamiolkowski et al., (1985) - #7

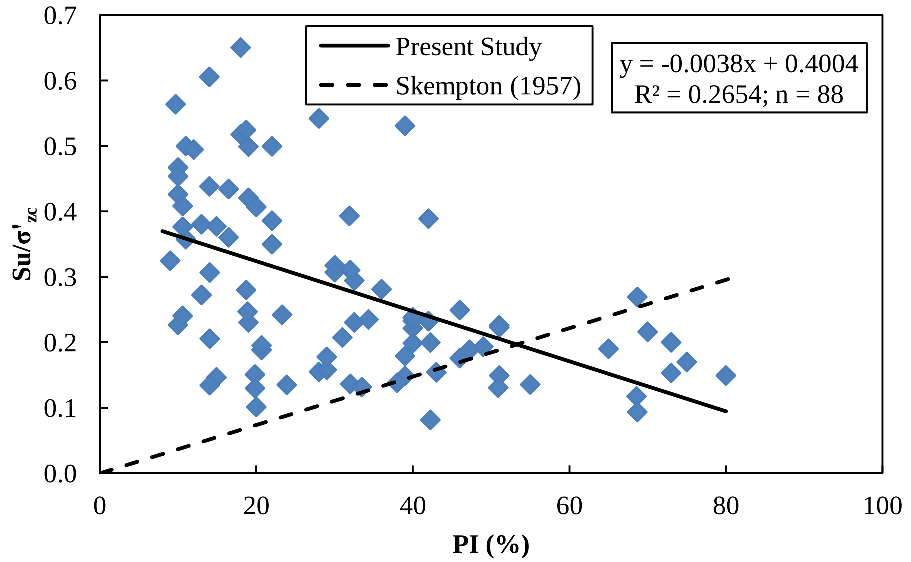


Figure 4.22: S_u (CU), Correlation Between S_u/σ'_{zc} and PI - Skempton (1957) - #8

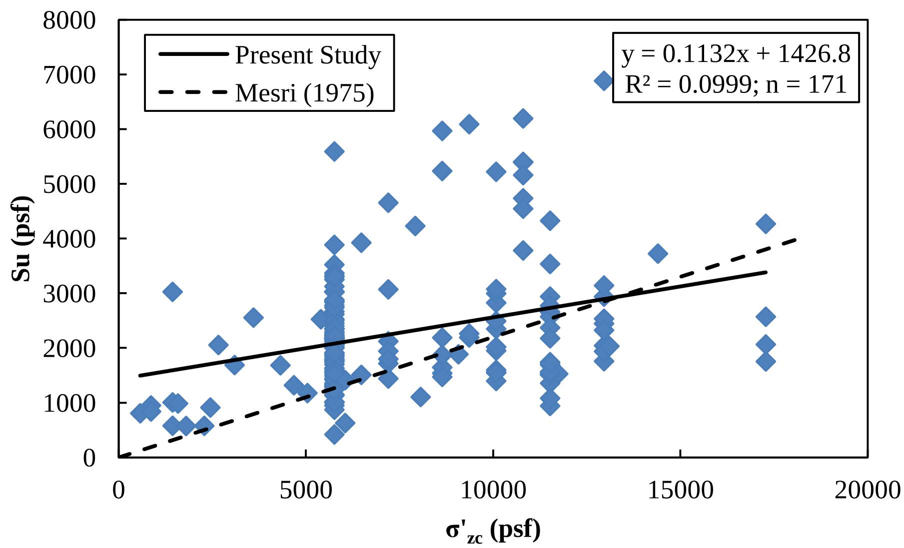


Figure 4.23: S_u (CU), Correlation Between S_u and σ'_{zc} – Mesri (1975) - #9

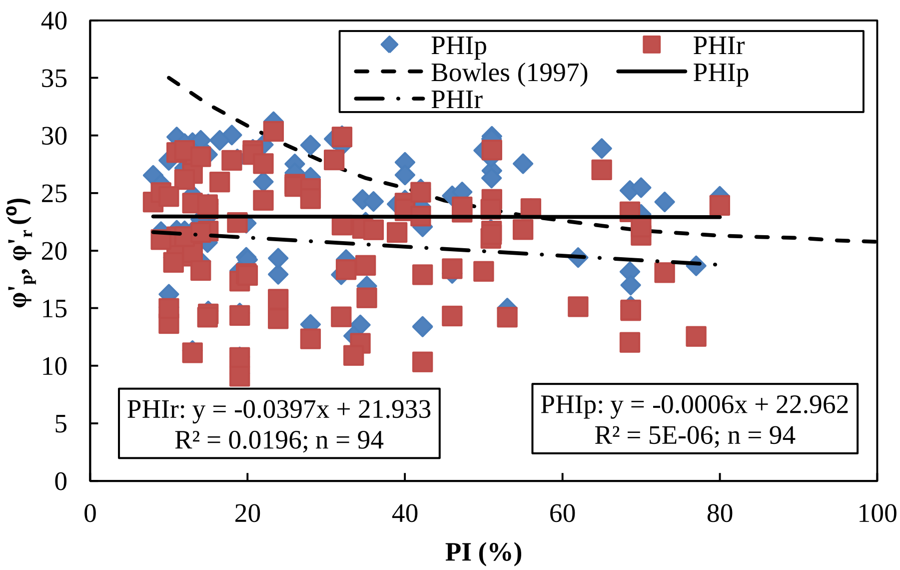


Figure 4.24: S_u (CU), Correlation Between φ'$_p$, φ'$_r$ and PI – Bowles (1997) - #10

Table 4.29: Consolidation Parameters – Cc and Cr - Published Empirical Correlations and Correlations Developed in this Study

#	Equation	Reference	Remarks
1	Cc = 0.009(LL - 10) Cc = 0.0066(LL - 10) + 0.0411	Skempton (1944) Present Study	R^2 = 0.506; n = 234
2	Cc = 0.40(e_o - 0.25) Cc = 0.4592(e_o - 0.25) – 0.0217	Azzouz et al., (1976) Present Study	R^2 = 0.7307; n = 256
3	Cc = 0.01(w_n - 5) Cc = 0.0133(w_n - 5) – 0.0679	Azzouz et al., (1976) Present Study	R^2 = 0.7633; n = 288
4	Cc = 0.37(e_o + 0.003LL – 0.34) Cc = 0.4121(e_o + 0.003LL – 0.34) – 0.0093	Azzouz et al., (1976) Present Study	R^2 = 0.7564; n = 230
5	Cc = 0.5G_s(PI/100) ≈ PI/74 Cc = 0.2803G_s(PI/100) + 0.0967	Wood and Worth (1978) Present Study	R^2 = 0.4383; n = 231
6	Cc = 0.00234 LLG_s Cc = 0.0024 LLG_s + 0.0254	Nagaraj and Murthy (1986) Present Study	R^2 = 0.506; n = 233
7	Cr = 0.15(e_o + 0.25) Cr = 0.0262(e_o + 0.007) + 0.0209	Azzouz et al., (1976) Present Study	R^2 = 0.1168; n = 199

Table 4.29 Cont'd: Consolidation Parameters – Cc and Cr - Published Empirical Correlations and Correlations Developed in this Study

#	Equation	Reference	Remarks
8	Cr = 0.003(w_n + 10) Cr = 0.0007(w_n + 7) + 0.0198	Azzouz et al., (1976) Present Study	R^2 = 0.0991; n = 201
9	Cr = 0.126(e_o + 0.003LL – 0.06) Cr = 0.0239(e_o + 0.003LL – 0.06) + 0.0216	Azzouz et al., (1976) Present Study	R^2 = 0.1276; n = 199
10	Cr = 0.00463 LLG_s Cr = 0.0001 LLG_s + 0.0303	Nagaraj and Murthy (1985) Present Study	R^2 = 0.0788; n = 165
11	Cr = Cc 1/5*Cc to 1/10*Cc Cr / Cc = 1/21.88* Cc	Budhu (2008) Present Study	R^2 = 0.0744; n = 197
Notes: e_0 = initial void ratio; w_n = natural water content; PI = plasticity index; LL = liquid limit; Gs = specific gravity of solids; Cc = compression index; Cr = recompression index			

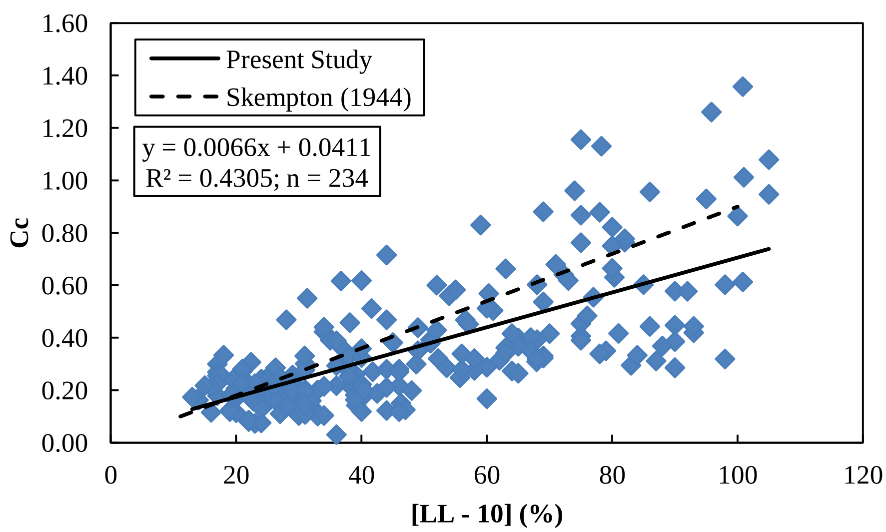


Figure 4.25: Consolidation, Correlation Between Cc and [LL - 10] - Skempton (1944) - #1

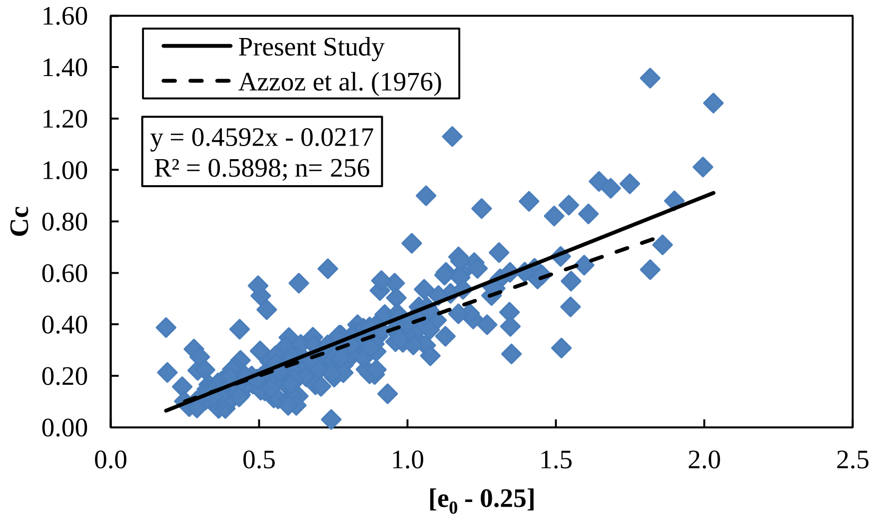


Figure 4.26: Consolidation, Correlation Between Cc and (e_0 - 0.25) - Azzouz et al., (1976) - #2

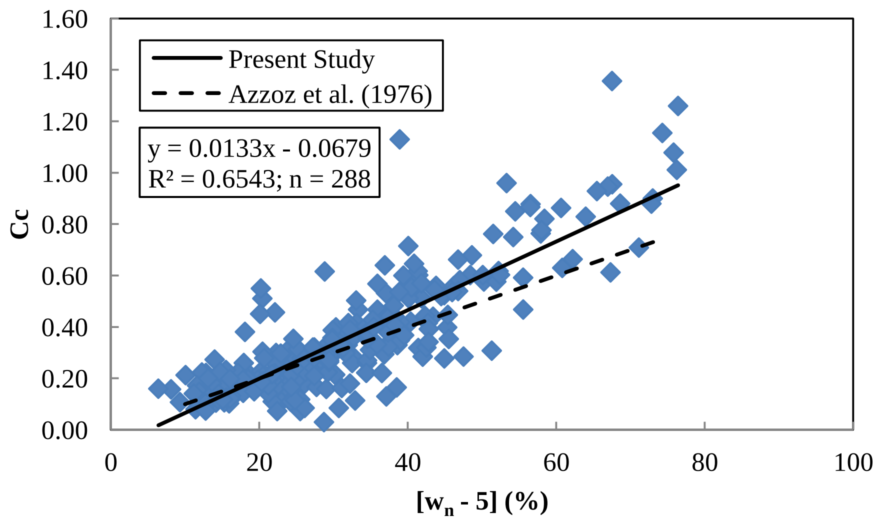


Figure 4.27: Consolidation, Correlation Between Cc and (w_n - 5) - Azzouz et al., (1976) - #3

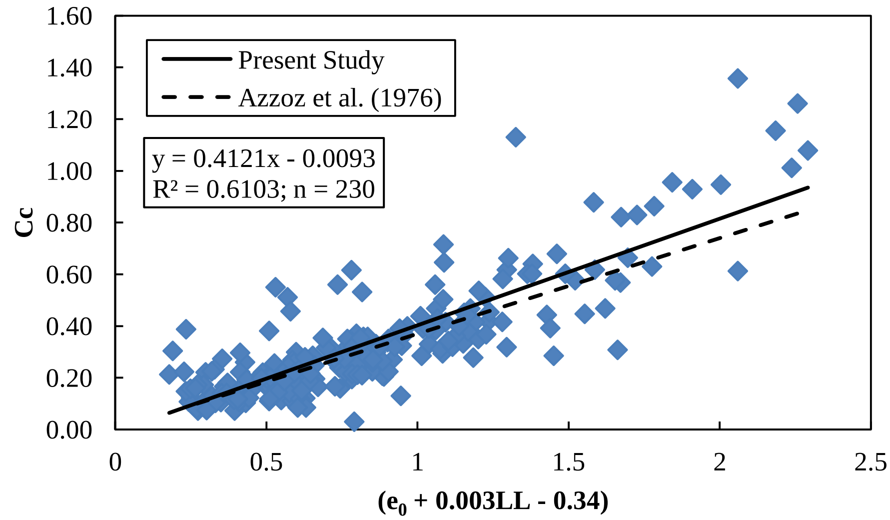


Figure 4.28: Consolidation, Correlation Between Cc and [e_0 + 0.003LL – 0.34] - Azzouz et al., (1976) - #4

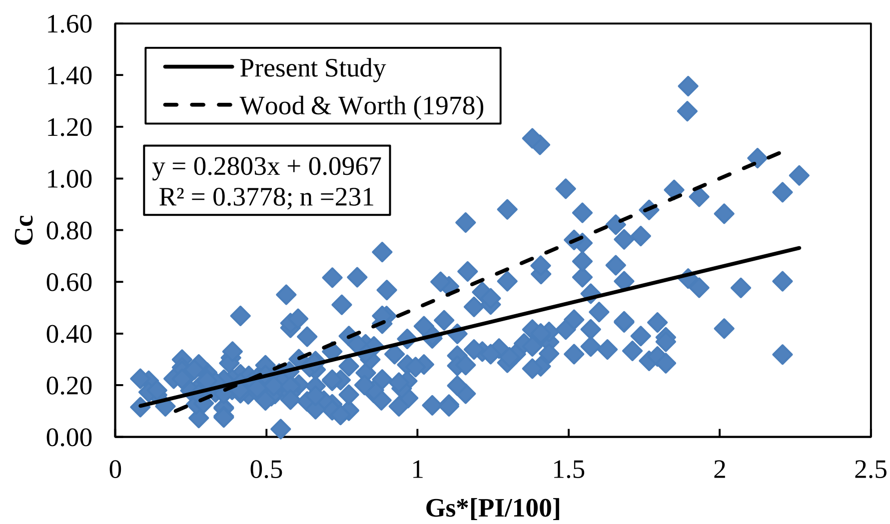


Figure 4.29: Consolidation, Correlation Between Cc and Gs*(PI/100) - Wood and Worth (1978) - #5

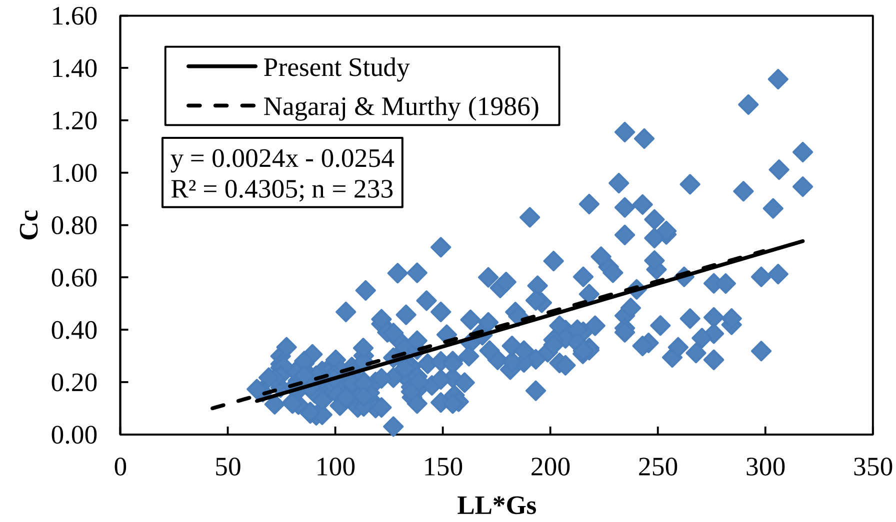


Figure 4.30: Consolidation, Correlation Between Cc and LL*Gs - Nagaraj and Murthy (1986) - #6

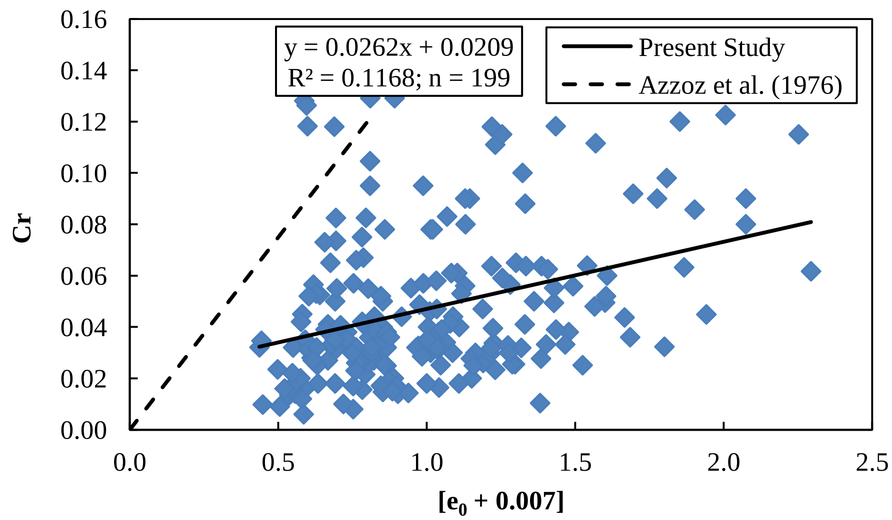


Figure 4.31: Consolidation, Correlation Between Cr and [e_0 - 0.007] - Azzouz et al., (1976) - #7

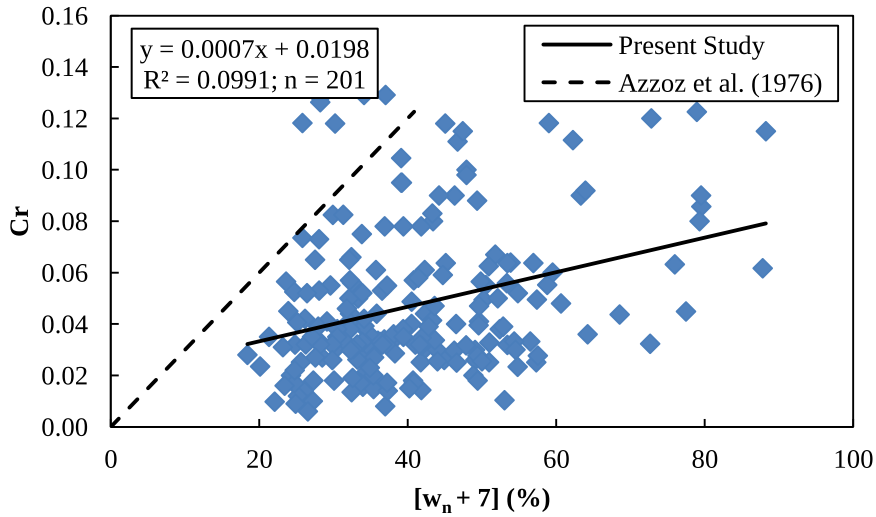


Figure 4.32: Consolidation, Correlation Between Cr and [w_n - 7] - Azzouz et al., (1976) - #8

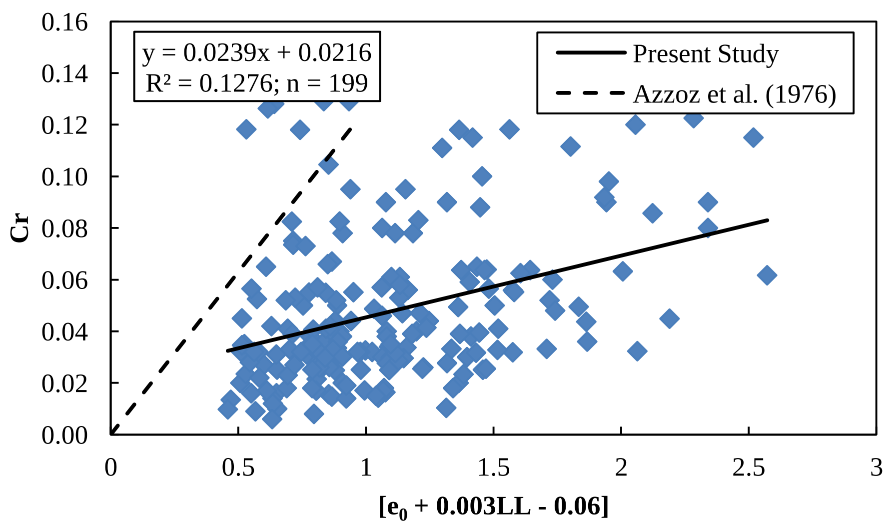


Figure 4.33: Consolidation, Correlation Between Cr and [e_0 + 0.003LL – 0.06] - Azzouz et al., (1976) - #9

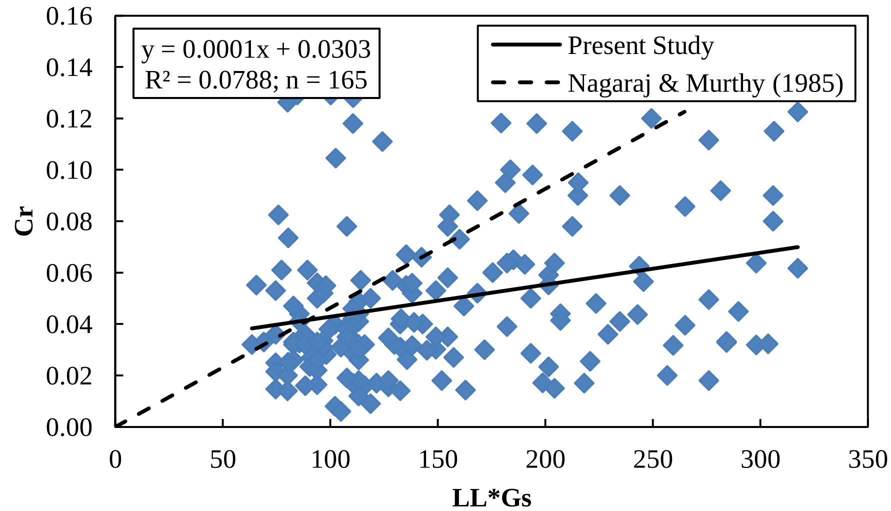


Figure 4.34: Consolidation, Correlation Between Cr and LL*Gs - Nagaraj and Murthy (1986) - #10

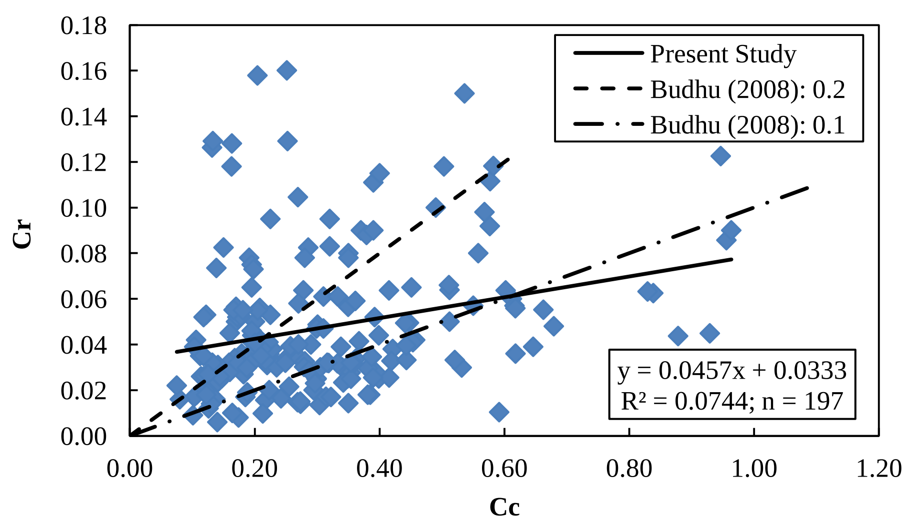


Figure 4.35: Consolidation, Correlation Between Cr and Cc - Budhu (2008) - #11

4.5. SPATIAL VARIABILITY OF PARAMETERS

4.5.1. General. In this study, the spatial variability of geotechnical parameters was primarily determined in terms of the scale of fluctuation, θ, using the semivariogram function (SVF). The scale of fluctuation defines the distances over which there is significant correlation of material property values. It is the third parameter required for the full characterization of the spatial variability of a geotechnical parameter; the first and second being the mean and the variance, respectively.

In this section, the scale of fluctuation of geotechnical parameters based on the research (S&T and MU) data were determined, the range of influence computed using Laboratory data and CPTu data was evaluated, and the scale of fluctuation determined using both the SVF and autocorrelation factor (ACF) were evaluated – in this case, the closely-, and uniformly-spaced CPTu data was used in this analysis. The ACF was described in § 2.2. In all cases, the spatial variability of the geotechnical parameters in the vertical direction was investigated.

4.5.2. Spatial Variability of Parameters. The SVF was adopted in this study because, compared to the ACF, the SVF has properties that makes it best suited for the spatial variability analysis of non-uniformly spaced data e.g. laboratory test data. It can also be used for the spatial variability analysis of closely-, and uniformly-spaced data. Unlike the ACF, the SVF does not require that data be spaced or obtained at equal intervals and the maximum number of lags allowable, K = N/2 (Kelkar and Perez, 2002), where N is the total number of data points. Hence with the SVF, a tolerance can be applied to the lag lengths to ensure that all the data points within the allowable range contribute to the analysis. Also, twice the number of data points in a dataset is used in the analysis compared to the ACF. With ACF, K = N/4 (Box and Jenkins, 1970; Anderson, 1976; Box et al., 1994; NIST, 2010).

The three most prevalent soil profile conditions encountered in the spatial variability analysis of geotechnical parameters in the vertical direction are the single-layer soil profile, the multi-layered soil profile, and the profile where there is a predominant soil layer and the other soil layers are not of interest. These conditions require the deployment of different measures for the spatial variability analysis to be successful. Following is a description of measures deployed in the spatial variability analyses of geotechnical parameters in these soil profile conditions:

- In the case of single-layer soil profile in the vertical direction, the scale of fluctuation of the parameter for the soil profile is evaluated using the autocorrelation function (ACF) or the semivariogram function (SVF).
- In the case of a multi-layered soil profile in the vertical direction, the scale of fluctuation of the parameter for each layer in the soil profile is evaluated using the autocorrelation function (ACF) or the semivariogram function (SVF). The average of the individual scales of fluctuation is reported as the scale of fluctuation of the parameter for the soil profile. The procedure was applied by Cherubini et al., (2007) in the characterization of some Italian soils for reliability analysis.
- In a soil profile where there is a predominant soil layer and the other soil layers are not of interest, the data from the other layers are not used in the determination of the scale of fluctuation of the parameter for that layer of the soil profile. This

method was adopted by Jaksa et al., (1997) in the modeling of the undrained shear strength of a stiff, overconsolidated clay (Keswick Clay) in Australia.

The procedure for the determination of the scale of fluctuation using the SVF is as follows:

1. Develop the profile for the parameter.
2. Validate the profile – check out for outliers
3. Develop the profile for the parameter.
4. Check the profile for the presence of any trend
    a. If a trend is present (dataset is nonstationary), it to a stationary one by removing a low-order polynomial trend, usually no higher than a quadratic, using the method of Ordinary Least Squares, OLS.
    b. Check stationarity by visual inspection of a scatter-plot of the data, and or the Kendal's τ test.
5. Compute and plot the experimental semivariogram using the lag as the distance (usually fixed) at which data is acquired. Use the original data in the computation if there is no trend present. Alternatively, if there is trend present use the residuals from the detrending process.
    a. For continuously acquired data at close, and regular intervals, use the Equation 23 below to compute the experimental variogram:

$$\gamma^{*}(h) = \frac{1}{2n(h)} \sum_{i=1}^{n(h)} [g(x_i) - g(x_i + h)]^2 \tag{23}$$

    b. For non-continuously acquired data at wide, and irregular intervals, use the Equation 24 below to compute the experimental variogram:

$$\gamma^* \,(h \pm \Delta h) = \frac{1}{2n(h \pm \Delta h)} \sum_{i=1}^{n(h)} [g(x_i) - g(x_i + h)]^2 \qquad (24)$$

6. Select a best-fit theoretical semivariogram model and fit it to the experimental semivariogram.
7. Determine the properties (the nugget effect, Co; the sill, C + Co; and the range of influence, *a*) of the fitted semivariogram model.
8. Select the relationship for the scale of fluctuation corresponding to the theoretical semivariogram model selected in (6).
9. Compute the scale of fluctuation using the relationship selected in (8).

The geostatistics software VESPER *version* 1.62 (Minasny et al., 2005), hereinafter referred to only as VESPER, was used in this study. VESPER is a PC-Windows program developed by the Australian Centre for Precision Agriculture (ACPA) at the University of Sydney, Australia, for spatial prediction that is capable of performing kriging with local variograms. The local variogram is modeled in the program by fitting a variogram model automatically through the nonlinear least-squares method. Several variogram models are available in the program, namely spherical, exponential, Gaussian and linear with sill. The characteristic parameters of the fitted variogram model are obtained and the range of influence, *a*, is used to determine the scale of fluctuation using the appropriate relationship (see Table 2.3 for some relationships). A sample screen shot and a plot showing the both the experimental variogram and the fitted theoretical (spherical) variogram from VESPER are presented in Figures 4.36 and 4.37, respectively. More detailed information on VESPER is presented elsewhere in VESPER (2010).

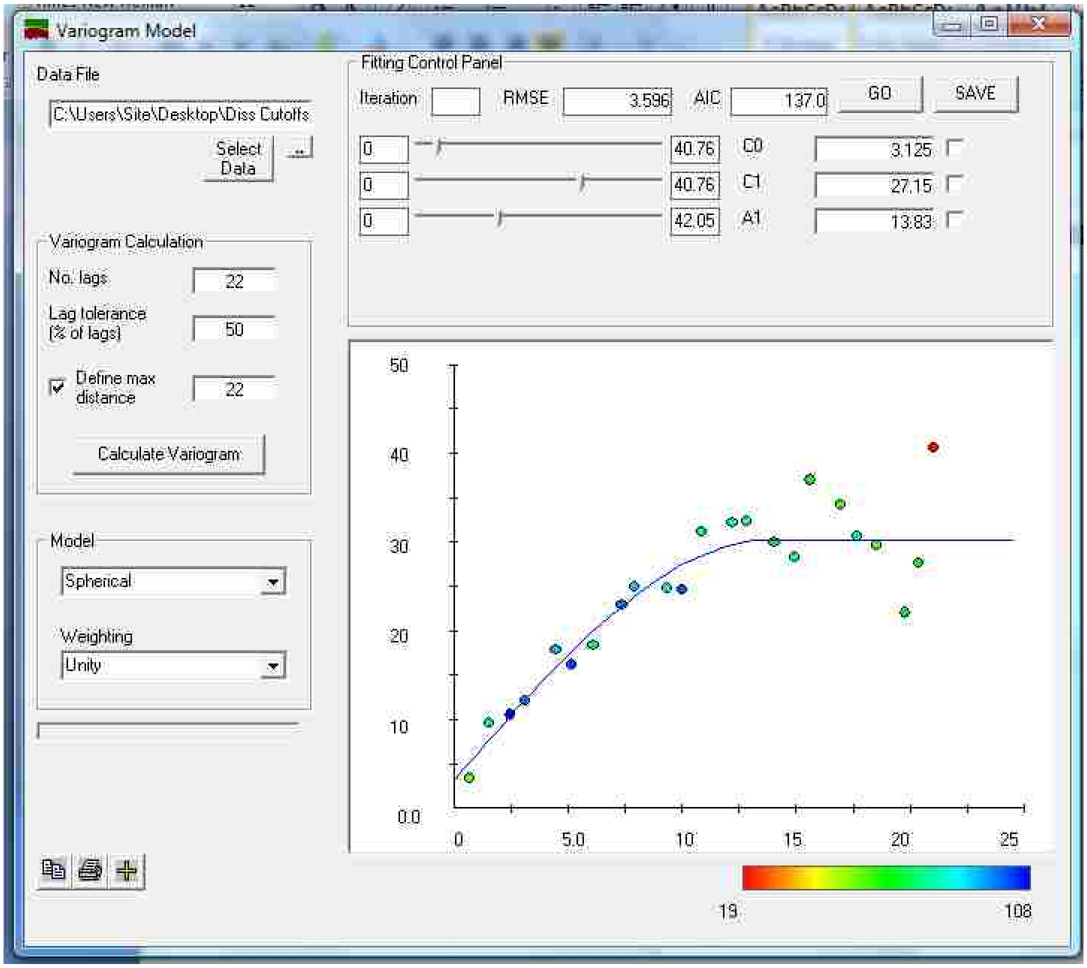


Figure 4.36: Sample Screen Shot from VESPER

For this analysis, the lag distance and the tolerance for the laboratory test data and the CPTu data were 1ft and 100%, and 0.16 ft and 25%, respectively. The lag distance for computing the experimental variogram is the distance interval at which data were acquired. The lag distance is approximately 1ft for the laboratory test data and 0.16 ft for CPTu data. The lag tolerance (percentage of lags) represents the percentage of the lag distance considered in the determination of the semivariogram. For example, 50% lag tolerance on a lag distance of 10 feet has a tolerance of 5 - 15 feet. A lag tolerance of

50% was applied to the irregularly-spaced laboratory data so as to take every data point into account in the determination of the semivariogram, while for the regular CPTu data, a smaller the lag tolerance of 25% was applied.

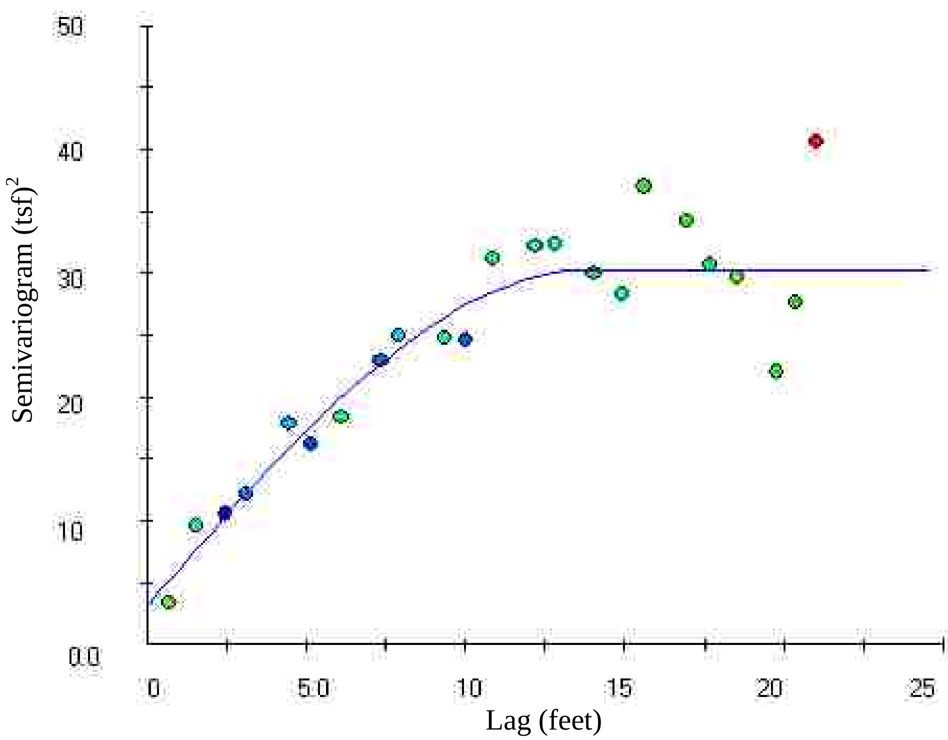


Figure 4.37: Experimental and the Fitted Theoretical (Spherical) Variogram (P-B7 PL)

The results of the spatial variability analyses are presented in Tables 4.30 and 4.31. Table 4.30a to 4.30d presents the scale of fluctuation values for some laboratory-tested properties for Warrensburg, St. Charles, New Florence, and Pemiscot, respectively. Table 4.31a to 4.31d presents the Scale of fluctuation values for some CPTu-tested properties for Warrensburg, St. Charles, New Florence, and Pemiscot, respectively. The laboratory-tested properties presented in Table 4.30a to 4.30d include compression index, Cc, initial void ratio, e_0, bulk unit weight, γ_b, liquid limit, LL, preconsolidation pressure, p'_c, friction angle, φ, plastic limit, PL, undrained shear strength (consolidated, undrained), S_u (CU), undrained shear strength (unconsolidated, undrained), S_u (UU), and natural water content, w_n. The CPTu-tested properties presented in Tables 4.31a to 4.31d include compression index, Cc, preconsolidation pressure, p'_c, undrained shear strength, S_u, cone resistance, q_c, and corrected cone resistance, q_t.

Tables 4.30 and 4.31 present the parameters of interest, the predominant soil classification type, the range and average value of the scale of fluctuation at each location. Also presented on Tables 4.30 and 4.31 are the values the range and average value of the scale of fluctuation from published references. The primary references are Jones et al., (2004) and Uzielli et al., (2007).

It should be noted that data presented in Uzielli et al., (2007) is a summary of data from other sources like Phoon et al., (1995), Kulhawy and Trautmann (1996), Lacasse and Nadim (1996), Phoon and Kulhawy (1999a), and Jones et al., (2002).

Table 4.30: Scale of Fluctuation Values for Some Laboratory-Tested Properties

Property	Soil Type	Range (ft)	Scale of Fluctuation [θ] (ft)	Published Values / Reference	
				Range (m)	Scale of Fluctuation [θ] (m)
(a) Warrensburg					
Cc	CL	2.5 - 3.3	2.9		
e_0	CL	1.6 - 2.0	1.8		3.0^+

Table 4.30 Cont'd: Scale of Fluctuation Values for Some Laboratory-Tested Properties

Property	Soil Type	Range (ft)	Scale of Fluctuation [θ] (ft)	Published Values / Reference	
				Range (m)	Scale of Fluctuation [θ] (m)
γ_b (pcf)	CL	1.9 - 2.3	2.1	2.4 - 7.9[*] / 1; 0.8 - 8.6[#] / 2	5.2[*] / 1
LL (%)	CL	3.0 - 3.2	3.1	1.6 - 8.7[*] / 1	5.2[*] / 1
p'_c (psf)	CL	4.4 - 4.7	4.6		0.6[+]
φ (^{o})	CL	1.1 - 1.1	1.1		
PL (%)	CL	1.8 - 1.9	1.9		
S_u [CU] (psf)	CL	1.8 - 1.8	1.8	0.8 - 6.1[#] / 1; 0.8 - 8.6[#] / 2	2.5[#] / 1
S_u [UU] (psf)	CL	2.5 - 2.9	2.7	0.8 - 6.1[#] / 1; 0.8 - 8.6[#] / 2	2.5[#] / 1
w_n (%)	CL	2.3 - 4.3	3.3	1.6 - 12.7[*] / 1	5.7[*] / 1
(b) St. Charles					
Cc	CH	-	-		
e_0	CH	1.2 - 2.7	2.0		3.0[+]
γ_b (pcf)	CH	2.2 - 5.1	3.7	2.4 - 7.9[*] / 1	5.2[*] / 1
LL (%)	CH	2.0 - 3.2	2.6	1.6 - 8.7[*] / 1	5.2[*] / 1
p'_c (psf)	CH	-	-		
φ (^{o})	CH	1.1 - 6.4	3.8		
PL (%)	CH	1.8 - 2.7	2.3		
S_u [CU] (psf)	CH	1.8 - 3.2	2.5	0.8 - 6.1[#] / 1; 0.8 - 8.6[#] / 2	2.5[#] / 1
S_u [UU] (psf)	CH	2.2 - 8.6	4.0	0.8 - 6.1[#] / 1; 0.8 - 8.6[#] / 2	2.5[#] / 1
w_n (%)	CH	1.2 - 4.7	3.0	1.6 - 12.7[*] / 1	5.7[*] / 1
(c) New Florence					
Cc	CL	-	-		
e_0	CL	2.3 - 3.1	3.4		3.0[+]
γ_b (pcf)	CL	1.5 - 2.3	1.9	2.4 - 7.9[*] / 1	5.2[*] / 1
LL (%)	CL	1.8 - 1.8	1.8	1.6 - 8.7[*] / 1	5.2[*] / 1
p'_c (psf)	CH	-	-		
φ (^{o})	CL	6.6 - 6.6	6.6		
PL (%)	CL	1.8 - 1.8	1.8		
S_u [CU] (psf)	CL	3.6 - 3.6	6.3	0.8 - 6.1[#] / 1; 0.8 - 8.6[#] / 2	2.5[#] / 1
S_u [UU] (psf)	CL	2.1 - 6.4	4.0	0.8 - 6.1[#] / 1; 0.8 - 8.6[#] / 2	2.5[#] / 1
w_n (%)	CL	2.5 - 12.0	5.7	1.6 - 12.7[*] / 1	5.7[*] / 1

Table 4.31: Scale of Fluctuation Values for Some CPTu-Tested Properties

Property	Soil Type	Range (ft)	Scale of Fluctuation [θ] (ft)	Published Values / Reference	
				Range (m)	Scale of Fluctuation (m)
(a) Warrensburg					
Cc	Clay / Silty Clay	1.3 - 4.5	2.9		
p'_c		2.6 - 7.8	5.2		
q_c		2.0 - 8.4	5.2	0.1 - 2.2[a] / 1	0.9[a] / 1
q_t		2.2 - 8.0	5.1	0.2 - 0.5[b] / 1	0.3[b] / 1
S_u		2.4 - 5.4	3.9		
(b) St. Charles					
Cc	Clay / Silty Clay	1.1 - 7.1	5.1		
p'_c		2.4 - 4.4	3.3		
q_c		1.9 - 4.0	2.8	0.1 - 2.2[a] / 1	0.9[a] / 1
q_t		2.3 - 3.9	3.0	0.2 - 0.5[b] / 1	0.3[b] / 1
S_u		2.2 - 3.9	3.0		
q_c	Sand	0.8 - 1.8	1.3		1.3 / 2
q_t		0.9 - 1.5	1.3		
(c) New Florence					
Cc	Clay / Silty Clay	1.0 - 2.8	4.8		
p'_c		3.8 - 7.8	5.5		
q_c		3.8 - 19.9	8.6	0.1 - 2.2[a] / 1	0.9[a] / 1
q_t		3.7 - 19.3	8.0	0.2 - 0.5[b] / 1	0.3[b] / 1
S_u		3.8 - 18.6	7.9		
(d) Pemiscot					
Cc	Clay / Silty Clay	12.7 - 21.9	17.3		
p'_c		22.5 - 35.4	29.0		
q_c		7.7 - 32.5	20.1	0.1 - 2.2[^] / 1	0.9[^] / 1
q_t		6.0 - 26.3	16.2	0.2 - 0.5[#] / 1	0.3[#] / 1
S_u		23.5 - 41.1	32.3		

Notes: 1 = Jones et al., (2004); 2 = Uzielli et al., (2007); [a] = Sand, Clay; [b] = Clay; Cc = compression index; p'_c = preconsolidation pressure; S_u = undrained shear strength; q_c = cone resistance; q_t = corrected cone resistance; 1ft = 0.3048m

4.5.3. Range of Influence - Laboratory Data Versus CPTu Data.

Investigations were led to compare of the range of influence, *a*, determined from continuous and regularly spaced CPTu data and the non-continuous and irregularly spaced laboratory test data from tests on Shelby tube sampling. The aim of these investigations was to identify the presence of any consistent relationship between the ranges of influence computed using both laboratory and CPTu data. The range of influence, a, is directly related to the scale of fluctuation, θ. Models exist that relate the range of influence to the scale of fluctuation, θ (Clark, 1979; Meek, 2001; Uzielli et al., 2007). The scale of fluctuation, θ, can also be determined analytically using equations stated in Cressie (1993).

The undrained shear strength profiles at four different locations (Warrensburg, St. Charles, New Florence, and Pemiscot) in Missouri were used for this analysis. The undrained shear strength, S_u (UU) determined from the unconsolidated undrained triaxial test (UU) on each specimen taken from Warrensburg, St. Charles, New Florence, and Pemiscot sites. The corresponding undrained shear strength from CPTu soundings taken at the immediate vicinity (about 2 to 3 feet) of each of the borehole locations was analyzed. Exploration at each location was completed within the two-week time frame suggested by Rethati (1988) for sampling (obtaining Shelby tube samples) and testing (CPTu) to be considered time-invariant.

The CPTu data were calibrated against laboratory test data, following one of the methods in Lunne et al., (1997), so that both results are comparable. The calibration of CPTu data against laboratory test data for this study was executed by Dr. Norbert Maerz and Kerry Magner both of the Field Testing Team of the TGRP. The calibration procedure, which principally aims at obtaining the appropriate cone factor, N_{kt}, for the CPTu data, was described previously in §4.2.

The bearing factor, N_{kt} values for the Warrensburg, St. Charles, New Florence, and Pemiscot sites were 6, 19, 15 and 15, respectively. The undrained shear strength profiles for both the laboratory and CPTu testing are presented in Figures 4.38a to 4.38d for the Warrensburg, St. Charles, New Florence, and Pemiscot locations, respectively.

Original data was used in the variogram analysis for Warrensburg, New Florence and Pemiscot while residuals from the removal of a linear trend were used for St. Charles. The removal of the best-fit linear trend from Warrensburg, New Florence and Pemiscot data resulted in residuals with stronger trend present. The lag distance for computing the experimental variogram is the distance interval at which data were acquired. Hence, for the laboratory data, the lag distance was 2 feet while for the CPTu data, the lag distance was 0.16 feet. Also for the CPTu data, for the purposes of comparing the two datasets and evaluating the effect of lag distance on range of influence, two more determinations at lag distances of 1- and 2-feet were implemented. The lag tolerance (percentage of lags) represents the percentage of the lag distance considered in the determination of the semivariogram. For example, 50% lag tolerance on a lag distance of 10 feet has a tolerance of 5 – 15 feet. A lag tolerance of 50% was applied to the irregularly-spaced laboratory data so as to take every data point into account in the determination of the semivariogram, while for the regular CPTu data, a smaller lag tolerance of 25% was applied. Also for the CPTu data, for the purposes of comparing the two datasets and evaluating the effect of lag tolerance on range of influence, two more determinations at lag tolerances of 50% and 0% were implemented. These variogram computation parameters are presented in Table 4.32.

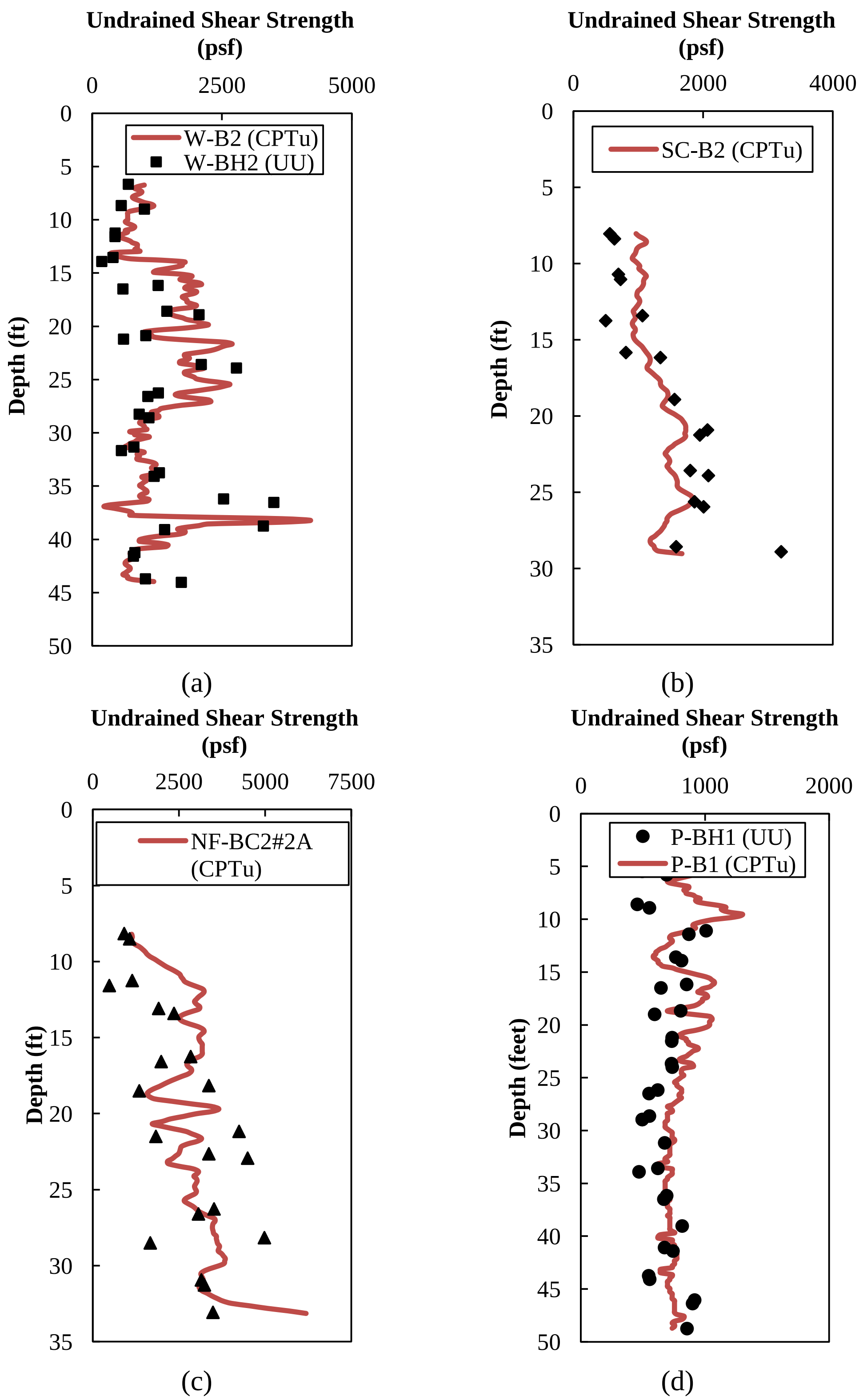


Figure 4.38: S_u (UU) Profile (a) Warrensburg (b) St. Charles (c) New Florence (d) Pemiscot

Table 4.32: Variogram Computation Parameters

Variable	Laboratory	Field (CPTu)				
Lag Distance (ft)	2	0.16	1	1	2	2
Tolerance (%)	50	25	50	0	50	0

The results of the variogram analysis for the Warrensburg, St. Charles, New Florence, and Pemiscot sites are presented in Figure 4.39 and Table 4.33. The laboratory data and field data for (lag = 0.16 ft, tolerance = 25%), and (lag = 2 ft, tolerance = 50%) in Figures 4.39a to 4.39d, and 4.39e to 4.39g, respectively. The variograms for other field data were omitted for purposes of clarity as they plotted very close to those for the field data in Figures 4.39e – 4.39g. A summary of parameters of the variograms are presented in Table 4.33. Comparisons of the effect of lag length and lag tolerance on the range of influence are presented in Figures 4.40a and 4.40b, respectively. The variograms were fitted on the basis of the lowest root-mean-square error. In Figure 4.39, variograms for the laboratory and the field data are denoted by borehole identifiers W-BH-, SC-BH-, NF-BH-, P-BH- and W-B-, SC-B-, NF-B-, P-B- for the Warrensburg, St. Charles, New Florence and Pemiscot sites, respectively. For the field data, the borehole identifiers are followed by the lag distance. In both cases, the letter, V, stands for the theoretical variogram. Hence, W-B2-0.16-V means theoretical variogram for Warrensburg, Boring 2, 0.16 feet lag distance. In Table 4.33, the letter, *z*, after the lag distance stands for 0% tolerance.

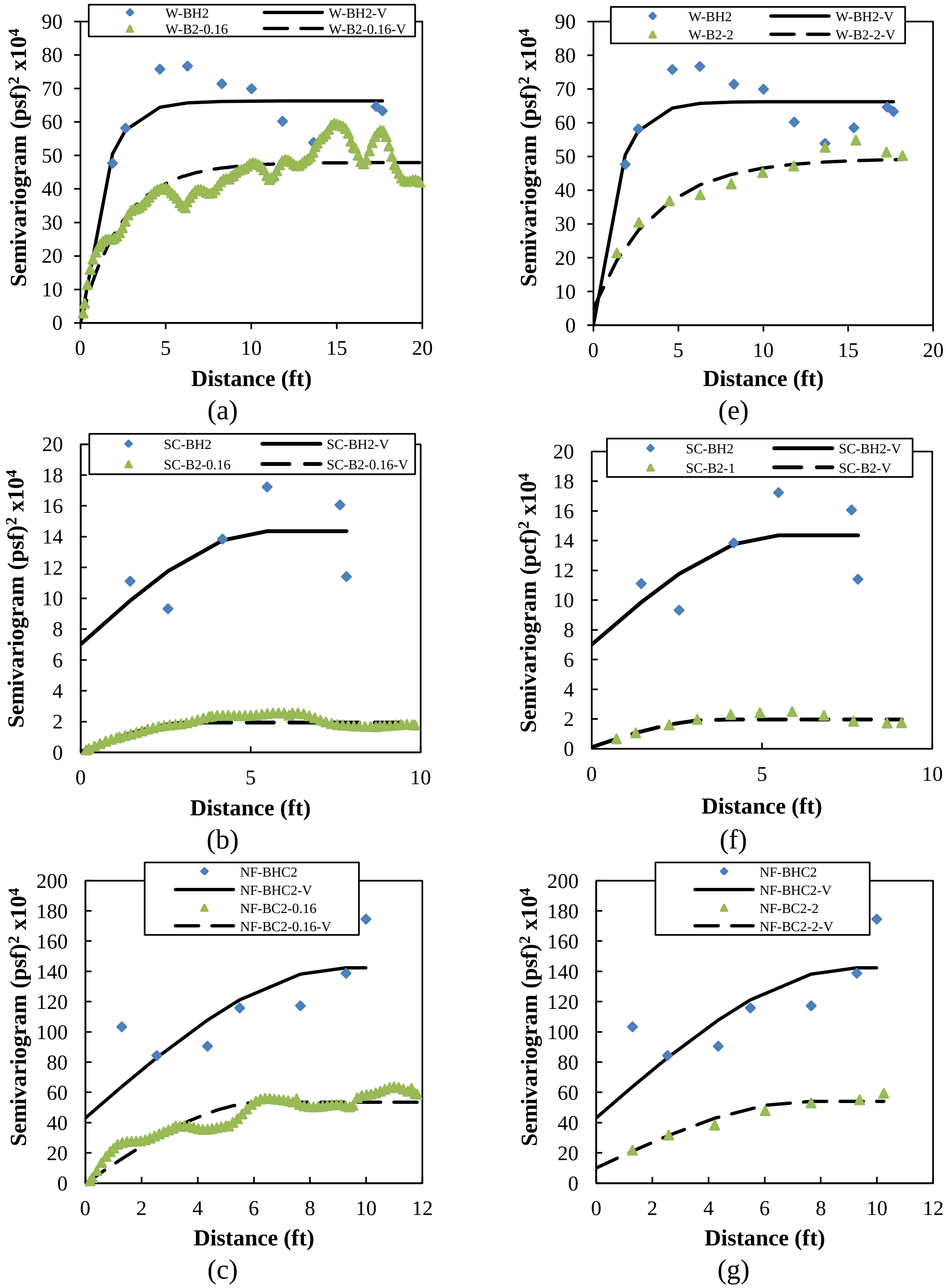
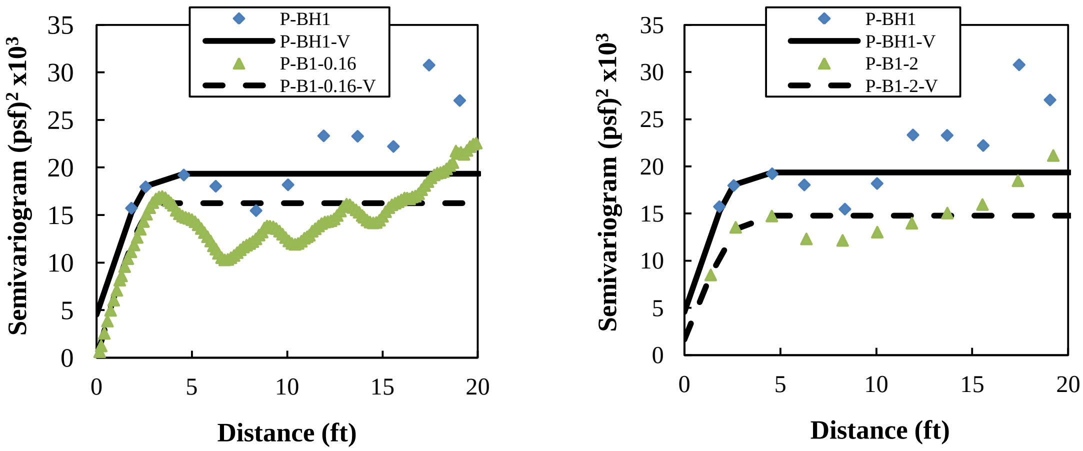


Figure 4.39: Semivariogram of Laboratory Data and CPTu Data



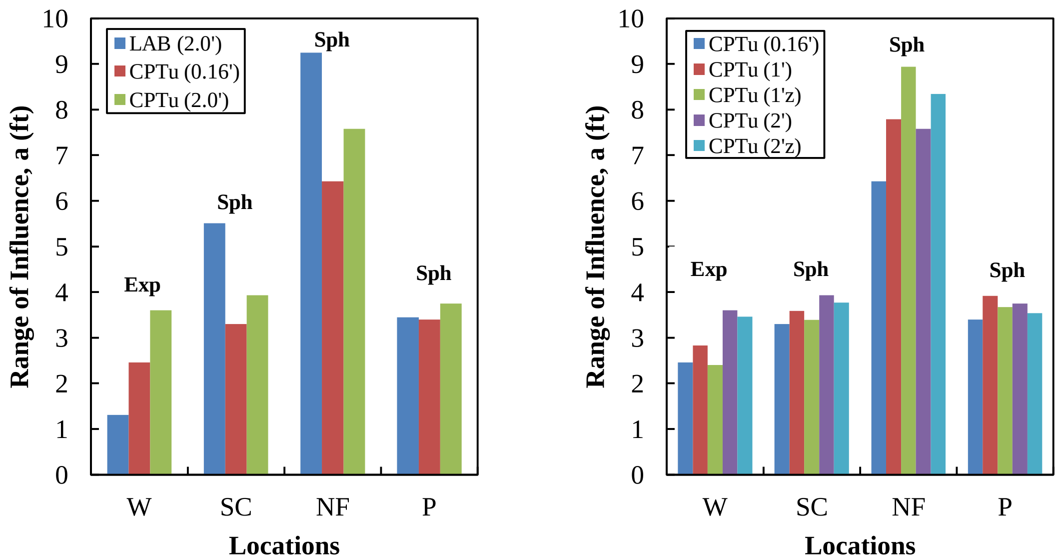
(d) (h)

Figure 4.39 Cont'd: Semivariogram of Laboratory Data and CPTu Data

Table 4.33: Variogram Parameters

Parameter	W-BH2	W-B2-0.16	W-B2-1	W-B2-1z	W-B2-2	W-B2-2z
Model	Exponential	Exponential	Exponential	Exponential	Exponential	Exponential
C_0 (psf)^2	0	595.8	50000	50000	50000	50000
C_1 (psf)^2	662494	478465	423764	420687	443733	445150
a (ft)	1.307	2.461	2.827	2.399	3.603	3.462
Parameter	SC-BH2	SC-B1-0.16	SC-B1-1	SC-B1-1z	SC-B1-2	SC-B1-2z
Model	Spherical	Spherical	Spherical	Spherical	Spherical	Spherical
C_0 (psf)^2	70114	413.7	1085.6	845.1	1573	1336
C_1 (psf)^2	73387	19027	18557	18405	18540	18577
a (ft)	5.505	3.299	3.585	3.393	3.925	3.767
Parameter	NF-BHC2	NF-BC2-0.16	NF-BC2-1	NF-BC2-1z	NF-BC2-2	NF-BC2-2z
Model	Spherical	Spherical	Spherical	Spherical	Spherical	Spherical
C_0 (psf)^2	431284	0	108311	88255	99395	110863
C_1 (psf)^2	991779	535777	437002	479810	441028	462651
a (ft)	9.25	6.425	7.79	8.943	7.578	9.34
Parameter	P-BH1	P-B1-0.16	P-B1-1	P-B1-1z	P-B1-2	P-B1-2z
Model	Spherical	Spherical	Spherical	Spherical	Spherical	Spherical
C_0 (psf)^2	4553.5	0	962.2	0	1657.9	1940.8
C_1 (psf)^2	14799	16233	15462	15888	13129	13451
a (ft)	3.452	3.396	3.920	3.668	3.751	3.541

Notes: NF-BHC2 = NF-BHC2#2A; NF-BC2 = NF-BC2#2A; C_0 = Nugget effect; C_1 = sill; a = range of influence (also, range)



(a) (b)

Figure 4.40: Variogram Parameters (a) Comparison of Lag Lengths, and (b) Comparison of Lag Tolerance

4.5.4. Scale of Fluctuation - Autocorrelation Function Versus Semivariogram Function. Investigations were led to compare of the scale of fluctuation, θ, determined using autocorrelation function and semivariogram function from continuous and regularly spaced CPTu data. The aim of these investigations was to identify the presence of any consistent relationship between the scales of fluctuation computed using both the autocorrelation and the semivariogram functions.With the autocorrelation function, the correlation length is determined while the range of influence, *a*, is determined with the semivariogram function. Both the correlation length and the range of influence are directly related to the scale of fluctuation. Models exist that relate the correlation length (Vanmarcke, 1983; Jaksa et al., 2000; Uzielli et al., 2007) and the range of influence (Clark, 1979; Meek, 2001; Uzielli et al., 2007) to the scale of fluctuation, θ.

The corrected cone resistance, q_t from seven soundings (P-B1 to P-B7) at the Pemiscot location was used for this analysis. The geostatistics software VESPER (Minasny et al., 2005) was used to determine the range of influence while Correlogram (Web-Reg, 2011), a Microsoft Excel add-in was used to determine the correlation length.

The corrected cone resistance, q_t, profiles for P-B1, P-B2, P-B3, P-B4, P-B5, P-B6 and P-B7 are presented in Figures 4.41a to 4.41g, respectively. In Figures 4.41a to 4.41g, the q_t profile for each sounding is divided in two parts: an upper part (A) and a lower part (B). The division in two parts was by a trial-and-error method. For the purposes of trend removal, best-fit trend lines (linear and quadratic) were fitted to selected sections of a profile, and the trend line which gave the highest coefficient of determination, determined by the Ordinary Least Squares method, for the section was selected for that section. The parts of the profiles which had high coefficients of determination, R^2, were detrended while the parts with low R^2 were not detrended. Both linear and quadratic trend lines were fitted. The trend lines fitted to each part of the profiles and their equations are presented in Table 4.34.

The residuals from the detrending process were used in both the autocorrelation and semivariogram analyses except for the upper parts of P-B1, P-B2, and the lower parts of P-B4, and P-B6, respectively where the original data were used. The lag distance used for both the autocorrelation and semivariogram analyses is the distance interval at which data were acquired. Hence, for the CPTu data, the lag distance was 0.16 feet. As the difference in spacing between data points is very small, the spacing was modified to multiples of 0.16 feet. Hence, a tolerance of 0% was applied.

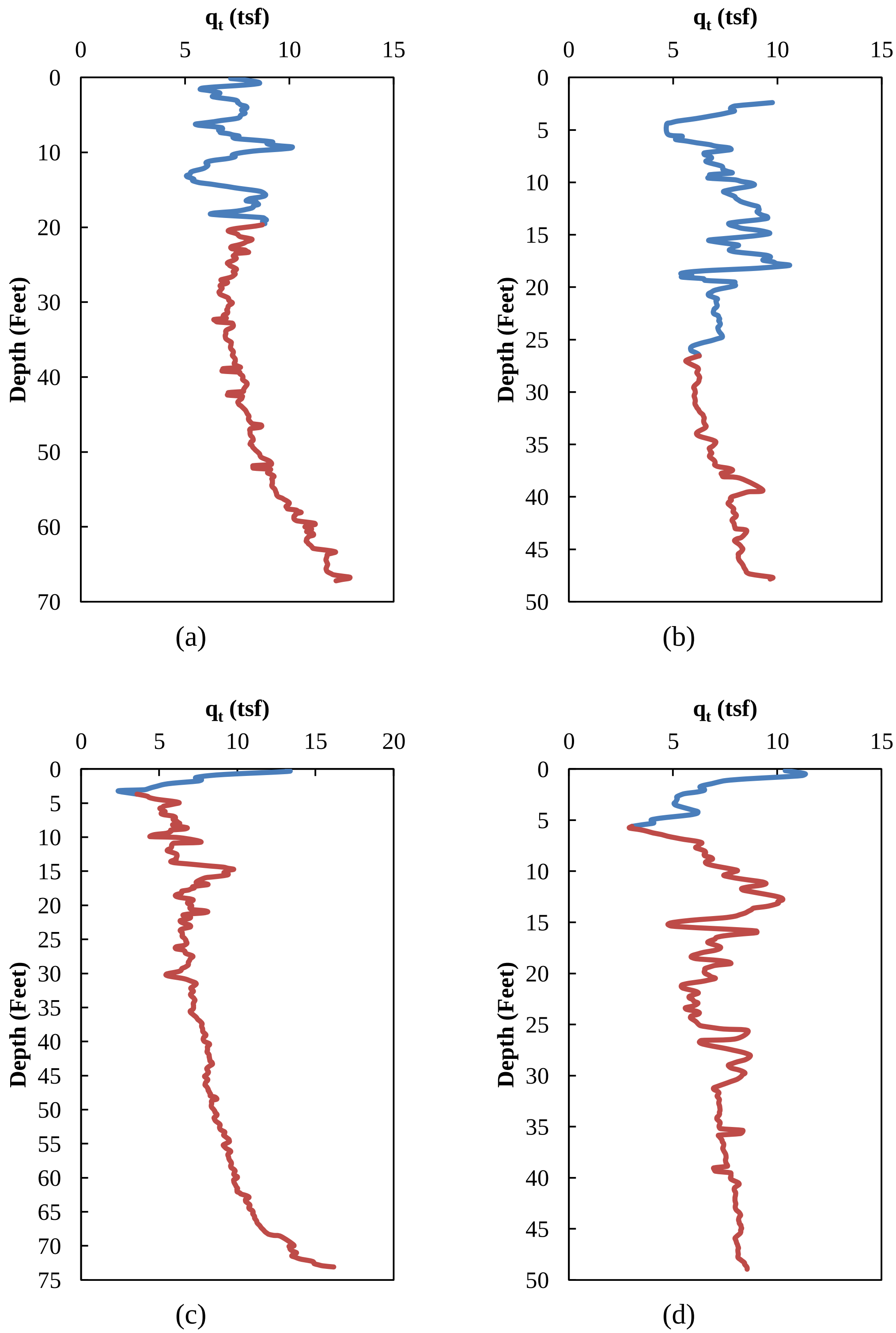
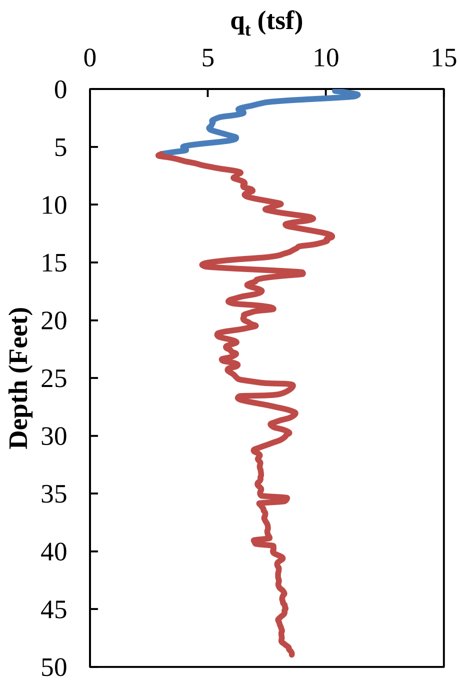
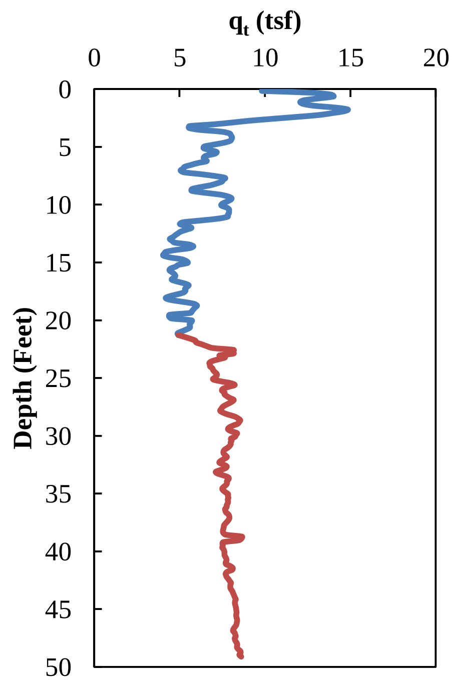


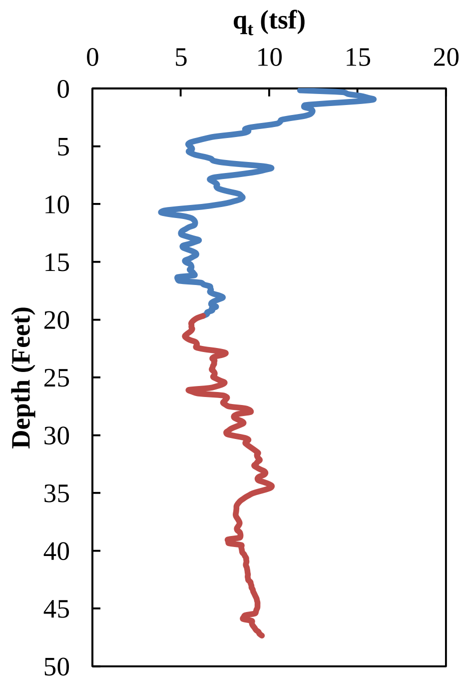
Figure 4.41: q_t Profiles (a) P-B1 (b) P-B2 (c) P-B3 (d) P-B4



(f)



(g)



(h)

Figure 4.41 Cont'd: q_t Profiles (c) P-B3 (d) P-B4 (e) P-B5 (f) P-B6 (g) P-B7 (h) P-B8

Table 4.34: Soundings and Equation of Best-Fit Lines

Sounding	Part	Equation: Best-fit line	R^2	Detrended
P-B1	A	$y = 0.6717x + 5.178$	0.0186	N
	B	$y = -0.9298x^2 + 24.909x - 97.866$	0.8015	Y
P-B2	A	$y = -0.1794x + 16.435$	0.0009	N
	B	$y = -0.9975x^2 + 20.866x - 59.594$	0.9020	Y
P-B3	A	$y = 0.0217x^2 - 0.6724x + 5.3308$	0.9473	Y
	B	$y = -0.7126x^2 + 21.579x - 86.709$	0.8195	Y
P-B4	A	$y = -0.668x + 7.1668$	0.7413	Y
	B	$y = 4.02x - 1.5799$	0.1309	N
P-B5	A	$y = 3.5041x - 13.324$	0.6132	Y
	B	$y = 9.4984x - 32.926$	0.8544	Y
P-B6	A	$y = 731.46x^{-2.412}$	0.6785	Y
	B	$y = 1.9234x^2 - 18.993x + 66.866$	0.4031	N
P-B7	A	$y = 750.6x^{-2.345}$	0.6172	Y
	B	$y = 2.427x^{1.259}$	0.6509	Y
Notes: N = No; Y = Yes				

To determine the scale of fluctuation using the semivariogram function, the experimental variogram was plotted and a best-fit model was fitted and the parameters of the model were determined. One of the model parameters, the range of influence, *a*, was then used to evaluate the scale of fluctuation using the appropriate expression from Table 2.3 as described previously in §2 (Literature Review). A sample of the semivariogram showing the experimental and the theoretical model fitted is presented in Figure 4.42.

For this study, the scale of fluctuation in the case of the autocorrelation function was determined as the point at which the autocorrelation curve crosses the Bartlett limits. The Bartlett limits method was adopted because of its simplicity. Other methods of determining the scale of fluctuation exist. One method which is similar to the method used for the semivariogram function involves selecting a best-fit model for the autocorrelation plot, obtaining the model parameters and using the correlation length (one of the model parameters) to compute the scale of fluctuation using the appropriate expression Table 2.4. A sample of the autocorrelation plot is presented in Figure 4.43. In Figure 4.43 the parallel dotted lines bestriding the horizontal axis indicates the Bartlett limits. The Bartlett limit was described in §2 (Literature Review).

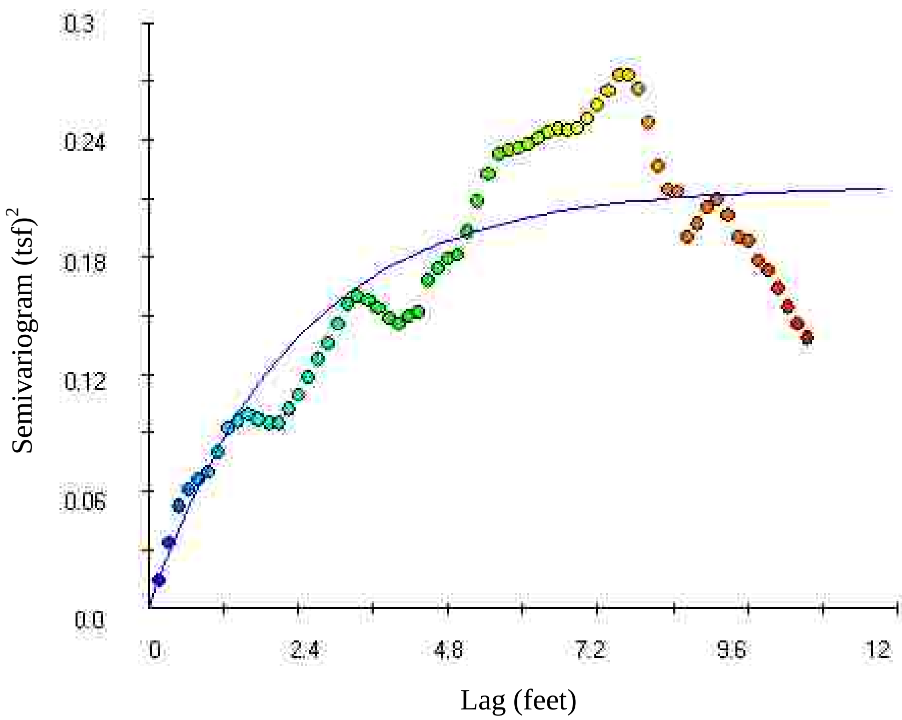


Figure 4.42: Sample Semivariogram Plot (P-B6A)

The results of the comparison of scale of fluctuation using autocorrelation function and semivariogram function are presented in Table 4.35 and Figure 4.44. Table 4.35 presents the scale of fluctuation for both parts of each profile and the average value for both the autocorrelation function and the semivariogram function. Also presented in Table 4.35 are theoretical models deployed in the semivariogram analyses. The average scale of fluctuation for the respective CPTu soundings computed using the autocorrelation function and the semivariogram function are presented in Figure 4.44.

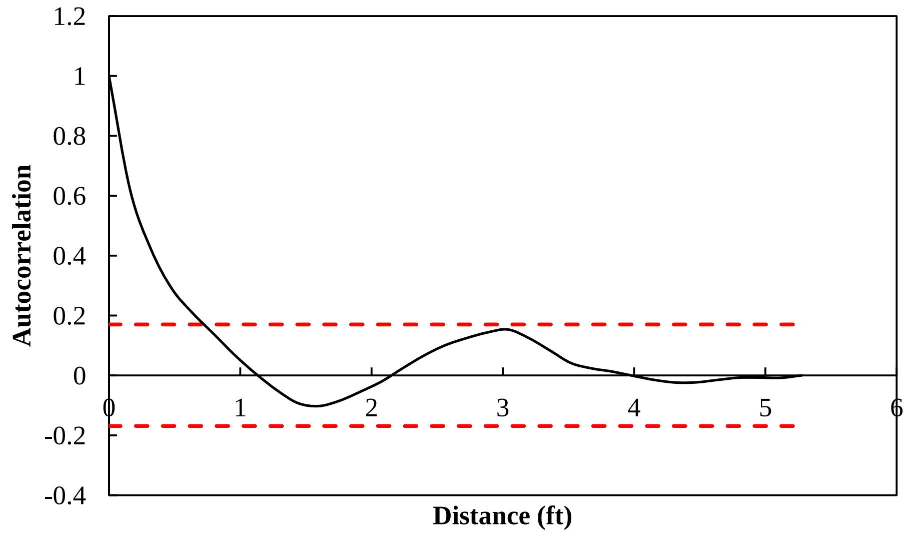


Figure 4.43: Sample Autocorrelation Plot (P-B6A)

Table 4.35: Range of Influence, Correlation Length, and Average Scale of Fluctuation of q_t

CPT Sounding	Part	ACF		SVF		
		θ (ft)	Ave θ (ft)	Model	θ (ft)	Ave θ (ft)
P-B1	A	1.35	4.28	S	1.96	7.29
	B	7.2		S	12.63	
P-B2	A	1.5	2	E	6.07	5.37
	B	2.5		E	4.68	
P-B3	A	0.16	4.58	S	0.95	4.76
	B	9		E	8.57	

Table 4.35 Cont'd: Range of Influence, Correlation Length, and Average Scale of Fluctuation of q_t

CPT Sounding	Part	ACF		SVF		
		θ (ft)	Ave θ (ft)	Model	θ (ft)	Ave θ (ft)
P-B4	A	0.84	2.12	S	2.42	2.99
	B	3.4		S	3.57	
P-B5	A	1.8	5.1	S	1.19	6.47
	B	8.4		S	11.75	
P-B6	A	0.75	2.08	S	1.34	4.35
	B	3.4		E	7.35	
P-B7	A	0.8	3.5	E	2.84	5.96
	B	6.2		E	9.08	

Notes: Ave = average; θ = scale of fluctuation; S = spherical; E = exponential; ACF = autocorrelation function; SVF = semivariogram function

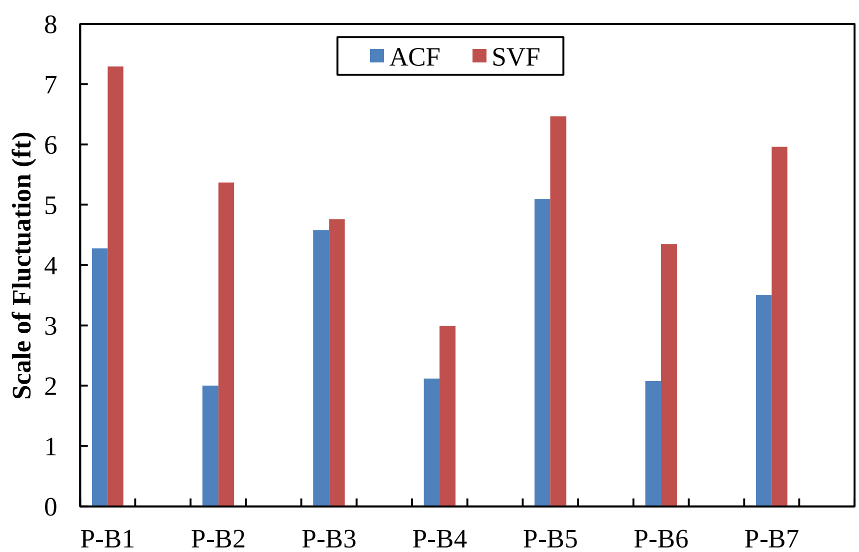


Figure 4.44: Average Scale of Fluctuation Computed Using the Autocorrelation Function and the Semivariogram Function

5. DISCUSSION

5.1. INTRODUCTION

The discussions of the results presented in Section 4 are presented in this section. The discussion of the result of the second moment statistics of geotechnical parameters, their probability distribution of geotechnical parameters, correlation between geotechnical parameters and the spatial variability of geotechnical parameters are presented below.

5.2. SECOND MOMENT STATISTICS: MEAN, VARIANCE, AND COEFFICIENT OF VARIATION (COV)

The COV of laboratory-determined parameters for Warrensburg, unclassified (both in terms of USCS soil type and in-situ state), and Warrensburg, classified (in terms of in-situ state only) and the COV of CPTu parameters for Warrensburg, classified (only in terms of soil type), and Warrensburg, classified (both in terms of soil type and in-situ state) were presented in Tables 4.3 to 4.8 and Tables 4.9a to 4.9c, respectively. A complete result of the second moment statistics is presented in Appendices C and D for laboratory-determined parameters and CPTu parameters, respectively.

From Tables 4.3 to 4.8 on laboratory-determined parameters, the COV are generally within the range published for the respective parameters with a few exceptions where the COV is much higher than the published values. The exceptional parameters are liquidity index, undrained shear strength, and preconsolidation pressure (combined CRS and IL).

There is a paucity of published data on the COV of parameters classified both in terms of USCS soil type and, or, in-situ state. Loehr et al., (2005) provides the only published data classified in terms of USCS soil type found. Parameters in Loehr et al., (2005) relevant to this study are unconfined compressive strength, plasticity index, natural moisture content, saturated unit weight, and direct shear strength parameters (φ, $\varphi_{c=0}$, and c'). Loehr et al., (2005) is main reference in Tables 4.6 to 4.8. The other two references are provided as a guide as they based on unclassified parameters.

The high COV values for liquidity index may be as a result of the higher operator bias in the determination of the Atterberg limits from which it is determined. Liquidity index values typically have a range of zero to one. However, for heavily overconsolidated soils which may have natural moisture content less than the plastic limit, a LI less than zero may be obtained. For sensitive clays which may have natural moisture content less than the liquid limit, a LI greater than one may be obtained (Das, 1998). Liquidity index values in the range between zero and one were used in the analyses. A very wide range of values of undrained shear strength was observed during laboratory testing. This must have resulted in the high COV values for undrained shear strength. The COV for preconsolidation pressure is due to the combination of values from both the Incremental Loading (IL) and the Constant Rate of Strain (CRS) tests. The CRS preconsolidation pressure values were sometimes found to be higher than the IL preconsolidation pressure. This may have contributed to the high COV values for preconsolidation pressure. The COV for consolidation parameters for each test type presented in Table 5.1 show these changes in the COV values.

In Table 5.1, the analysis of consolidation test data by test type show that compared to the CRS, IL yielded a higher COV for all parameter in Warrensburg and Pemiscot. For St. Charles, IL had a higher COV for Cc and Cr and a lower COV for preconsolidation pressure. While for New Florence, there was generally no difference in COV.

Table 5.1: COV (%) for Consolidation Parameters (Locations) - Constant Rate of Strain (CRS), Incremental Loading (IL), and (ALL = CRS+IL) Tests

Type	Location	Cc	Cr	$p'_{p\text{-}T}$	$p'_{p\text{-}C}$	$p'_{p\text{-}SE}$
ALL	Warrensburg	74	114	57	51	46
CRS		16*	30*	53*	59*	47*
IL		91	129	57*	52	48*
ALL	St Charles	44	332	50	37	47
CRS		20	39	54	50	44
IL		54	266*	29*	27	24*
ALL	New Florence	45	55	132	115	106
CRS		40*	54*	61*	59*	41*
IL		35	51	60*	39	47*
ALL	Pemiscot	49	53	40	46	37
CRS		39	51	25	25	22
IL		53	53	67	49	36
ALL	KCiCON	48	62	-	42	-
CRS		-	-	-	-	-
IL		48	62	-	42	-
Reported COV (%) / Reference		10-37/1; 73/2		10-35/1; 62/2	10-35/1; 62/2	10-35/1; 62/2

Notes: * = data count less than 20; Cc = compression index; Cr = recompression index; $p'_{c\text{-}T}$ = preconsolidation pressure (Tangent); $p'_{c\text{-}C}$ = preconsolidation pressure (Casagrande); $p'_{c\text{-}SE}$ = preconsolidation pressure (Strain Energy); CRS = constant rate of strain; IL = incremental loading; 1= Uzielli et al., (2007); 2 = Corotis et al., (1975)

The classification of parameters in terms of both USCS soil type and in-situ state resulted variously in a reduction in the COV for most soil classification types or in-situ states and an increase the COV for a few others. This is shown in Tables 5.2 and 5.3 for some parameters. Tables 5.2 and 5.3 present the COV (Locations) for parameters both unclassified (ALL) and USCS-classified and both unclassified (ALL) and in-situ state-classified, respectively. For the USCS-classified, only data on the more preponderant CL and CH soil types are presented. In most cases, other USCS soil types were present but in fewer numbers.

This change in COV as a result of further refinement by way of classification in terms of either or both of USCS soil type and in-situ state followed no particular pattern. The change will be as a result of the interplay of factors including the number and values of the parameter in each group.

CPTu data are presented according to soil type. Therefore, only the effect of in-situ state on the second moment statistics was investigated. There is a paucity of published data on the COV of both measured and estimated CPTu parameters. No published data was found on estimated CPTu parameters; this is expected as estimated CPTu values are dependent on the empirical relationships used in their determination. q_c and q_t for clay and sand were compared to published COV data. The COV for q_c was, in all cases, within the range of published values while that for q_t was above published values by about 10 to 20 percentage points.

Further refinement, by way of classification in terms of in-situ state, of CPTu data resulted in change in the COV of parameters similar to that observed with the laboratory data. Further classification in terms of in-situ state resulted variously in a reduction in the COV of parameters for most in-situ states and an increase the COV for a few others. This change is shown in Table 5.4 for some parameters.

COV like other second moment statistics of geotechnical engineering parameters is largely dataset-dependent, which data distribution is largely dependent on soil type and in-situ state. Hence there is a need to include both soil type and in-situ state when reporting COV values for geotechnical engineering parameters.

Table 5.2: COV (%) for Some Laboratory Parameters (Locations) – Unclassified (ALL) and USCS-Classified

Soil Type	Location	w_n	e_0	PL	γ_b	S_u (UU)	S_{up} (CU)	φ'_r
ALL	Warrensburg	15	15	21	5	95	65	28
CL		14	14	19	5	70	65	28
CH		23*	8*	14*	2*	-	-	-
ALL	St Charles	22	22	23	8	49	56	25
CL		19	24	16	7	12*	27*	25*
CH		21	20	19	7	70*	30*	27
ALL	New Florence	21	24	17	9	50	70	30
CL		21	24	17	7	50	66	30
CH		17	21	15	6	-	74*	29*
ALL	Pemiscot	27	26	22	11	22	27	29
CL		17	22	10	6	32*	9	6*
CH		26	23	21	11	22	25	26
ALL	KCiCON	28	25	31	10	23	90	17*
CL		28	24	17	9	-	82	8*
CH		13	9	36	5	29	28*	23*
Reported COV (%) / Reference		8-30/1	7-30/1; 41.5/2	6-30/1; 29/2	<10/1	10-30/1	20-40/1	5-15/1

Notes: * = data count less than 20; 1= Uzielli et al., (2007); 2 = Corotis et al., (1975); w_n = natural water content; e_0 = initial void ratio; γ_b = bulk unit weight; PL = plastic limit; S_u (UU) = undrained shear strength (unconsolidated, undrained); S_{up} (CU) = peak undrained shear strength (consolidated, undrained); φ'_r = effective friction angle (residual)

Table 5.3: COV (%) for Some Laboratory Parameters (Locations) – Unclassified (ALL) and In-Situ State-Classified

In-Situ State	Location	w_n	e_0	PL	γ_b	S_u (UU)	S_{up} (CU)	Φ'_r
ALL	Warrensburg	15	15	21	5	95	65	28
AGWL		19	18	16	6	-	-	-
BGWL		11	14	20	5	70	66	28
ALL	St Charles	22	23	23	8	49	56	25
AGWL		23	13	18	6	99	7*	1
BGWL		20	22	24	7	47	58	25
ALL	New Florence	21	24	17	9	50	70	30
AGWL		20	23	16	9	50	70	30
BGWL		17*	8*	24	4*	-	-	-
ALL	Pemiscot	27	26	22	11	22	27	29
AGWL		16*	-	14	-	-	-	-
BGWL		27	26	22	11	22	27	29
ALL	KCiCON	28	25	31	10	28	90	17
AGWL		28	25	34	10	26	90	17
BGWL		25	16	10	7*	31*	-	-
Reported COV (%) / Reference		8-30/1	7-30/1; 41.5/2	6-30/1; 29/2	<10/1	10-30/1	20-40/1	5-15/1

Notes: * = data count less than 20; 1= Uzielli et al., (2007); 2 = Corotis et al., (1975); w_n = natural water content; e_0 = initial void ratio; γ_b = bulk unit weight; PL = plastic limit; S_u (UU) = undrained shear strength (unconsolidated, undrained); S_{up} (CU) = peak undrained shear strength (consolidated, undrained); φ'r = friction angle (residual)

Table 5.4: COV (%) for Some CPTu Parameters (Locations) – Unclassified (ALL) and In-Situ State-Classified

Location	Soil Type	In-Situ State	q_c	f_s	q_t	S_u	σ'_{vo}	γ	q_t/σ'_{vo}
Warrensburg	Clay	ALL	51	73	48.5	84.9	41.3	6.6	60.6
		AGWL	54	64	52.6	49.9	42.3	5.3	55.4
		BGWL	74.1	74.7	70.8	100.8	29.7	6.9	91.6
	Clay+Silty Clay	ALL	68.1	107.8	66.3	79.8	15.9	7.8	68.9
		AGWL	-	-	-	-	-	-	-
		BGWL	68.1	107.8	66.3	79.8	15.9	7.8	68.9
St Charles	Clay	ALL	31.3	40.5	31.1	27.7	49.5	4.2	127.2
		AGWL	11	21	9.2	10.1	26.1	3.6	135.5
		BGWL	30.4	42.3	30	26.3	40.8	4.2	125.6

Table 5.4 Cont'd: COV (%) for Some CPTu Parameters (Locations) – Unclassified (ALL) and In-Situ State-Classified

Location	Soil Type	In-Situ State	q_c	f_s	q_t	S_u	σ'_{vo}	γ	q_t/σ'_{vo}
St Charles	Clay+Silty Clay	ALL	48.2	47	35.5	31.7	38.5	4.7	146.1
		AGWL	36.2	24.1	23.5	24.1	29.2	4.6	4.9
		BGWL	45.4	48.9	32	27.6	24	4.3	136.1
New Florence	Clay	ALL	43.4	38.1	42.1	43.1	55	3.6	33.2
		AGWL	69.9	58.4	64.9	82.5	46.1	7.1	7.5
		BGWL	38.1	35	36.7	37.1	48.2	2.8	29.7
	Clay+Silty Clay	ALL	34.5	39.5	34.5	38.4	45.9	4.5	26.5
		AGWL	39.7	56.4	39.4	46.8	24.1	7.2	6.7
		BGWL	26.9	33.6	26.8	33.8	35.2	3.4	22.3
Pemiscot	Clay	ALL	23.7	32.3	24.3	23.8	47.6	2.2	37.3
		AGWL	25.7	24.1	26	25.5	25.9	2.1	-
		BGWL	23.4	30.4	24	23.6	43.4	2.1	37.5
	Clay+Silty Clay	ALL	46.4	65.9	60.7	66.8	38.9	6.6	120.5
		AGWL	-	-	-	-	-	-	-
		BGWL	46.4	65.9	60.7	66.8	38.9	6.6	120.5

Notes: q_c = Cone Resistance; f_s = Side Friction; q_t = Corrected Cone Resistance; S_u = Undrained Shear Strength; σ'_{vo} = effective overburden pressure; γ = unit weight; q_t/σ'_{vo} = Normalized Corrected Cone Resistance

5.3. PROBABILITY DISTRIBUTION

5.3.1. Probability Distribution Types. The probability distribution of laboratory-determined parameters for Warrensburg, unclassified (both in terms of USCS soil type and in-situ state), and Warrensburg, classified (in terms of in-situ state only) and the probability distribution of CPTu parameters for Warrensburg, classified (only in terms of soil type), and Warrensburg, classified (both in terms of soil type and in-situ state) were presented in Tables 4.11 to 4.16 and Tables 4.17a to 4.17c, respectively. A complete result of the second moment statistics is presented in Appendices F and G for laboratory-determined parameters and CPTu parameters, respectively.

From Tables 4.11 to 4.17 on the probability distribution of both laboratory- and CPTu-determined parameters, there are cases where the fitted Pearson's type and the published distributions are in agreement. There was no agreement between fitted and published distribution in most cases. Instances of cases where both the fitted and published distributions are in agreement for the unclassified laboratory data include w_n, PL, S_u (UU), and e_0 among others. Similar cases where both fitted and published distributions are in agreement are also present in the classified laboratory data and classified CPTu data.

The results from the probability distribution analysis indicate that several other types of distributions other than the most prevalent three: normal, lognormal, and beta can be employed with geotechnical parameters. This was found to be the case in with both lab test data (with few data points) and the CPTU data (with many data points). It should be noted that the equivalent of the three most used distributions in geotechnical engineering literature; normal, lognormal, and beta on the Pearson's space are normal [$(\beta_1,\beta_2) = (0,3)$], Type VI, and Type I, respectively. Other Pearson's types that have the shape of the normal distribution are the Type I($\cap$) and Type IV while for the lognormal distribution it is Type I($\cap$).

5.3.2. Effect of Soil Classification Type and In-Situ State on Probability Distribution. Further refinement of data by grouping according to USCS soil type and in-situ state for the laboratory test data and in-situ state for the CPTu data resulted in change in the distribution type similar to that observed with COV in §5.2. The change in COV as a result of further refinement of data by way of classification in terms of either or both of USCS soil type and in-situ state followed no particular pattern. This change in distribution type due to data refinements is presented in Tables 5.5 and 5.6. The changes in distribution types are also present in Table 4.10 and Figure 4.1 for qt and S_u (UU) and in Tables 4.18 and 4.19 and Figure 4.2 for Su (UU).

Table 5.5: Distribution Types for Some Laboratory Parameters (Locations) – Unclassified (ALL) and USCS-Classified

Location	Soil Type	PI	LI	e_0	S_u (UU)	S_{up}	Cc	Cr
Warrensburg	ALL	IV	I(∩)	IV	I(J)	I(J)	I(∩)	I(J)
	CL	IV	I(∩)	IV	I(J)	I(J)	I(∩)	I(J)
	CH	I(U)	-	-	-	-	-	-
St Charles	ALL	I(∩)	I(J)	I(∩)	III	VI	III	I(∩)
	CL	I(∩)	I(∩)	VI	I(∩)	I(∩)	I(J)	-
	CH	I(∩)	I(J)	I(∩)	VI	I(J)	III	I(J)
New Florence	ALL	II	I(J)	IV	I(∩)	I(J)	I(J)	I(J)
	CL	I(∩)	I(J)	IV	I(∩)	I(J)	III	I(J)
	CH	I(∩)	I(∩)	I(∩)	-	-	I(J)	I(J)
Pemiscot	ALL	I(∩)	I(∩)	I(∩)	I(∩)	I(∩)	I(∩)	I(∩)
	CL	I(J)	I(U)	I(J)	I(∩)	I(J)	I(J)	-
	CH	N	I(∩)	I(∩)	I(∩)	I(∩)	I(∩)	I(J)
KCiCON	ALL	I(J)	I(∩)	I(U)	I(∩)	I(J)	I(∩)	I(∩)
	CL	I(∩)	I(U)	I(J)	-	I(J)	I(J)	I(J)
	CH	I(∩)	I(∩)	I(∩)	I(∩)	I(U)	IV	IV

Notes: PI = plasticity index; LI = liquidity index; e_0 = initial void ratio; S_u (UU) = undrained shear strength (unconsolidated, undrained); S_{up} (CU) = peak undrained shear strength (consolidated, undrained); Cc = compression index; Cr = recompression index

Table 5.6: Distribution Types for Some CPTu Parameters (Locations) – Unclassified (ALL) and In-Situ State-Classified

Location	In-Situ State	q_c	f_s	q_t	S_u	σ'_{vo}	γ	q_t/σ'_{vo}
Warrensburg	NONE	I(∩)	I(J)	I(J)	I(U)	I(∩)	I(∩)	I(J)
	AGWL	I(J)	I(J)	I(J)	I(∩)	I(J)	I(∩)	I(U)
	BGWL	VI	I(J)	VI	I(J)	I(U)	I(∩)	I(J)

Table 5.6 Cont'd: Distribution Types for Some CPTu Parameters (Locations) – Unclassified (ALL) and In-Situ State-Classified

Location	In-Situ State	q_c	f_s	q_t	S_u	σ'_{vo}	γ	q_t/σ'_{vo}
St Charles	NONE	I(∩)	I(∩)	I(∩)	I(∩)	I(U)	IV	I(U)
	AGWL	IV	IV	IV	II	I(∩)	I(U)	-
	BGWL	I(∩)	I(∩)	I(∩)	I(∩)	I(U)	I(∩)	I(U)
New Florence	NONE	IV	IV	IV	IV	I(U)	VI	I(U)
	AGWL	I(J)	I(U)	I(U)	I(J)	I(J)	I(U)	I(∩)
	BGWL	IV	IV	IV	VI	I(U)	IV	I(U)
Pemiscot	NONE	IV	VI	IV	VI	I(∩)	IV	I(J)
	AGWL	IV	IV	IV	IV	I(∩)	IV	-
	BGWL	VI	VI	IV	IV	I(∩)	IV	I(J)
Notes: q_c = Cone Resistance; f_s = Side Friction; q_t = Corrected Cone Resistance; S_u = Undrained Shear Strength; σ'_{vo} = effective overburden pressure; γ = unit weight; q_t/σ'_{vo} = Normalized Corrected Cone Resistance								

5.4. CORRELATION OF PARAMETERS

The results of the Correlation of Parameters are discussed hereinafter under the following headings: Correlation Matrix; Correlation between Normalized Undrained Shear Strength and PI; and Comparison of Study Data to Existing Empirical Relationships.

5.4.1. Correlation Matrix. The results of studies, based on the research (MU and S&T) data, into the development of correlations matrices between parameters were presented in Tables 4.20 to 4.23.

Correlation matrices are largely dataset-dependent and therefore cannot really be compared to other published correlation matrices. Within the context of RBD, the coefficients of correlation between parameters is required by those design approaches that takes into account both the uncertainty of the parameters and their correlation structure like that proposed by Low (2005).

It is also important to point out the effect of the coefficients of correlation of certain parameters as they relate to the development of empirical correlations between simply-obtained and complexly-obtained parameters, and the assessment of the validity of existing empirical correlations carried out in this study. For the location data (Tables 4.20 to 4.23), the following trends can be identified:

- The weak correlation between Atterberg limits with Su (UU) and Su (CU). The effect of this is the low quality empirical correlations developed between (PI) overburden stress normalized undrained shear strength (S_u/σ') and plasticity index in §5.4.3.1.

In this case, the coefficients of correlation between these parameters gave an indication of the quality of empirical correlations developed between them.

5.4.2. Correlation Between Normalized Undrained Shear Strength and PI. The results of studies, based on the research (MU and S&T) data, into the development of correlations between overburden stress-normalized undrained shear strength (S_u/σ') and plasticity index (PI) were presented in Figures 4.3 to 4.4, and Table 4.24.

Initial analysis based on unclassified data for each site (see Figures 4.3 to 4.6) shows that for the Warrensburg and Pemiscot sites data had a wide scatter; hence a wide degree of variability. The best fit curve for the correlation model was linear for Warrensburg and quadratic for Pemiscot. In each case the quality of the correlation model was low with coefficient of determination less than 0.2. For the New Florence and St. Charles sites, the best fit curve for the correlation model was quadratic for Pemiscot polynomial correlations were achieved with mid-range coefficients of determination of 0.68 and 0.47, respectively. Back calculating for S_u using equations from Skempton (1957), Worth and Houlsby (1985) and Jamiolkowski et al., (1985) consistently resulted in smaller values of S_u in each case suggesting that these previous empirical correlations are a little bit conservative compared to this study.

Further analyses, based on data pooled together from all four locations and grouped according their USCS soil types, were carried out with the aim of obtaining better correlation models for each soil type. These analyses produced a linear correlation model for CH soils and quadratic models for both CL, and ML soils. However, like in the unclassified, site-based analyses, the coefficients of determination of these correlation models were low: 0.48, 0.37, and 0.39 for CL, CH, and MH, respectively.

Results from this study show the relationship between S_u/σ' and PI to be largely nonlinear. They have been found to be largely quadratic in this case. This finding is in contradistinction to most published correlations between S_u/σ' and PI (Skempton and Henkel, 1953; Skempton, 1957; Karlsson and Viberg, 1967; Worth and Houlsby, 1985; Chandler, 1988) which are largely linear. This is an indication of how imprecise estimates made uncritically on the basis of published correlations can be and makes the case for the deployment of only appropriate correlations in estimations. Also, the low coefficients of determination of the correlation models developed in this study is indicative of the difficulty in predicting with any high level of certainty S_u on the basis of only Atterberg limits. Factors that influence S_u include clay content, sample disturbance, elevation of ground water table, geological condition, chemical and mechanical degradation (Matsuo and Shogaki, 1988; Yin, 1999; Trauner et al., 2005). Other soil properties that have significant effect on S_u and hence will be very helpful in its prediction include the effective overburden pressure (considered in this study), the unit weight, and the OCR.

5.4.3. Assessment of the Validity of Published Empirical Relationships Between Parameters. The discussions on the results of the Assessment of the validity of published empirical relationships between parameters are presented following. For purposes of clarity, the results are discussed in terms of strength correlations, and consolidation correlations.

5.4.3.1. Strength correlations. The results of the analyses to assess the validity of published empirical correlations relating strength parameters to simply-obtained parameters were presented in Figures 4.10 to 4.24 and Tables 4.27 to 4.28.

For the correlations with S_u (UU), the validity of nine correlations was assessed. Of the nine correlations assessed, only in one case ($S_u/\sigma'_{zc} = 0.22$ – Mesri, 1975) was a similar (to the correlation being assessed) slope obtained. However, the intercept in this one case was higher. Of the other eight correlations, only in one case [$S_u/\sigma'_z = (0.23 \pm 0.04)*OCR^{0.8}$ – Jamiolkowski et al., 1985] was the direction of the slope of the best-fit line from this study and slope of the correlation being assessed the same; positive in this case. In the other seven cases, both slopes were in the opposite directions with the slope of the correlation being assessed negative. The coefficient of determination in all cases was exceptionally low; less than 0.15.

For the correlations with S_u (CU), the validity of 10 correlations was assessed. Of the 10 correlations assessed, in five cases [$S_u/p'_o = 0.11 + 0.0037I_P$ – Skempton and Henkel (1953), Skempton (1957); $S_u/p'_o = 0.129 + 0.00435PI$ – Worth and Houlsby

(1985); $S_u/\sigma'_z = (0.23 \pm 0.04)*OCR^{0.8}$ – Jamiolkowski et al., 1985; $Su/\sigma'_{zc} = 0.22$ – Mesri, 1975] a similar (to the correlation being assessed) slope but with higher intercept was obtained. For the other five correlations, the direction of the slope of the best-fit line from this study and slope of the correlation being assessed were in the opposite directions with the slope of the correlation being assessed negative. The coefficient of determination in all cases was exceptionally low; less than 0.1 with one exception ($S_u/\sigma'_{zc} = 0.11 + 0.0037PI$ – Skempton, 1957) which had a coefficient of determination of 0.2654.

On the basis of the plots and the coefficient of determination of the fitted models, it can generally be concluded that the published empirical strength correlations assessed are not valid for the data in this study. This conclusion applies to both the overburden stress normalized undrained shear strength (S_u/σ') and preconsolidation stress normalized undrained shear strength (S_u/σ'_{zc}) relationships These empirical strength correlations primarily relate Atterberg limits to normalized S_u. As discussed in §5.4.2 it can be difficult to predict Su on the basis of Atterberg limits alone since a number of other factors influence S_u. Also, some of these correlations were developed from tests on soils which are not similar to the soils encountered in this study. For instance, Worth and Houlsby (1985) was based on tests on offshore soils from the North Sea and the Gulf of Mexico, Skempton and Henkel (1953) was based on postglacial estuary clays in London, Karlsson and Viberg was based on Swedish clays, and Bjerrum and Simons (1960) and Skempton (1957) were based on widely sourced data from England, Norway, New Zealand, and Sweden. In some other cases, a different type of test was employed. The correlation in Mesri (1975) is based on vane shear tests.

5.4.3.2. Consolidation correlations. The results of the analyses to assess the validity of published empirical correlations relating consolidation parameters to simply-obtained parameters were presented in Figures 4.25 to 4.35, and Table 4.29.

For the correlations with Cc, the validity of six correlations was assessed. Of the six correlations assessed, only in one case (Cc = 0.5Gs*(PI/100) – Wood and Worth, 1978) was the slope of the correlation obtained not close to the slope of the correlation being assessed. The other five correlations plotted quite closely to the correlation being assessed and the coefficients of determination of the correlations were high; in the range 0.43 to 0.65.

For the correlations with Cr, the validity of five correlations was assessed. In each case the best-fit line from this study had a positive slope similar to that of the correlation being assessed. However for all cases, the slope from this study was steeper than that of correlation being assessed. The correlations developed from this study were of a low quality with coefficients of determination of the correlations in the range 0.07 to 0.13.

The lack of fit of the study data to the Cr correlations may be due to the changes in the structure of the soil as a result of the application of compressive forces during consolidation testing. This change in the structure of the soil may affect how the soil rebounds thereby introducing a high degree of variability in Cr. This in turn could possibly have contributed to the lack of fit of study data to Cr correlations.

5.5. SPATIAL VARIABILITY

The results of Spatial Variability are discussed hereinafter under the following headings: Scale of Fluctuation; Comparison of Scale of Fluctuation computed with different Data Types (Laboratory, and Field - CPTu); and Comparison of Scale of Fluctuation Computed Following the Time Series/Random Field Theory and the Geostatistics Approach.

5.5.1. Scale of Fluctuation. The results of the analyses to determine the scale of fluctuation were presented in Tables 4.30 and 4.31.

The scale of fluctuation of parameters computed from laboratory testing data was generally within the range (at the lower end) of published values where they are available. There is a paucity of published data on the scale of fluctuation of parameters computed from laboratory testing data. For the parameters computed from CPTu data, the scale of fluctuation was greater than published values in all cases. Published values are available for measured parameters (q_t, and q_c in this case) only. There are no published values to compare the scale of fluctuation of estimated parameters to as they are dependent on the empirical correlation used in their determination and hence subject to change.

The scale of fluctuation determined here were for parameters in the predominant soil classification types (CH, CL). For the other less frequently occurring soil types like (CL-ML and CH-MH) where, due to paucity of data, scale of fluctuation determinations cannot be carried out, a combination of expert opinion and engineering judgment will be required to determine suitable values.

The scale of fluctuation of geotechnical engineering parameters is largely dataset-dependent hence these values determined here should not be applied uncritically to other situations.

5.5.2. Range of Influence - Laboratory Data Versus CPTu Data. The results of the variogram analyses to study the scale of fluctuation computed with different data types – laboratory data and field (CPTu) data were presented in Table 4.33 and Figure 4.40.

In all cases, the sill (= variance) of the variograms of the field data plotted below that of the laboratory data. This can be attributed to the much larger number of field data which must have led to a reduced variance. For the exponential model (fitted to the Warrensburg data), the smaller slope at the origin for the field data resulted in higher values of range than that for the laboratory data. While for the spherical model (fitted to the St. Charles, the New Florence, and Pemiscot data), the smaller slope at the origin for the field data can result in higher or lower values of range depending on the distance at which the sill is reached. In this case, the values of the range are generally lower for the field data. The increase of the value of the range for Warrensburg field data and the Pemiscot field data are in the region of 200 to 300% and 10 to 15% of the laboratory data value, respectively. The decrease in the value of the range for the St. Charles and the New Florence field data are in the region of 30 to 40% and 5 to 30%, respectively.

Comparing the range of influence for both datasets at 50% tolerance and 2-foot lag distance, the range of influence for the field data was higher for Warrensburg (about 300%) and Pemiscot (10%) and lower for St. Charles (about 30%), and the New Florence (about 20%) sites. The removal of tolerance (50% to 0%) from the field data led to a decrease in range for Warrensburg, St. Charles and Pemiscot, and an increase for New Florence. The magnitude of the change in range both cases were small.

There are differences in the range of influence computed using the two different datasets of interest. These differences, in terms of increase or decrease, follow no particular pattern but may be dependent on the type of variogram model employed. For the exponential model in which the semivariogram reaches the sill asymptotically and the range of influence is the lag distance where the semivariogram reaches the 95% of the sill value, the combination of a lower slope at origin and lower sill resulted in a higher range of influence for the field test data. For the spherical model in which the semivariogram reaches the sill value at lag distance of *a*, which is the range of influence, the combination of a lower slope at origin and lower sill resulted in a lower range of influence for the field test data.

It is not possible to draw broad conclusions on the basis of the limited studies reported herein. Considering both number of data and the quality of profile obtained, the range of influence (hence the scale of fluctuation) is best determined using CPTu data. However the use of CPTu is not prevalent in industry. Hence, it is important that that the relationship between the range of influence from CPTu data and the range of influence from other more prevalent sampling and testing methods as amended is determined. The development of this relationship will be of particular importance to practitioners who do not have other types of data (not CPTu) and want to employ the spatial averaging effect in reliability-based design. Hence, further studies on this subject involving more boreholes and more semivariogram models are recommended.

5.5.3. Scale of Fluctuation - Autocorrelation Function Versus Semivariogram Function. The results of the comparison of scale of fluctuation using autocorrelation function and semivariogram function were presented in Table 4.35 and Figure 4.44.

For the constituent parts of each sounding, the scale of fluctuation, θ, computed using the SVF was mostly higher than that computed using the ACF. The scale of fluctuation computed using the ACF was higher than that computed using the SVF only in two out of 14 cases. For each sounding, the average scale of fluctuation computed using the SVF was higher than that computed using the ACF. The magnitude of this difference ranged from 4 to 270%.

While the method employed in the determination of scale of fluctuation with the ACF is not dependent on the model fitted, the method employed in the determination of scale of fluctuation with the SVF can be model-dependent. For the ACF, the Bartlett's limit method (Jaksa et al., 2000; Uzielli et al., 2007) where the scale of fluctuation is taken as the distance at which the autocorrelation curve cuts the Bartlett's limit was employed. In the case of the SVF, the range of influence, and hence the scale of fluctuation was determined with the aid of the best-fit model which is selected on the basis of the lowest root-mean-square-error (VESPAR, 2011). This method of determination of scale of fluctuation by SVF was employed in this study to ensure consistency.

The variability of soil properties averaged over a domain is less than that of their point properties (Vanmarcke, 1977). This is known as the averaging effect in spatial variability. The spatial averaging effect results in a reduction in the variance used in RBD. To take advantage of the spatial averaging effect, the scale of fluctuation has to be determined. The scale of fluctuation is then used to compute the variance reduction factor (VRF) which is the factor by which the point property of a parameter is reduced. With the scale of fluctuation constant, the VRF depend on the averaging length: the longer the averaging length, the lower the VRF.

For a given averaging length, an increase in the scale of fluctuation leads to a reduction in the VRF and hence a greater reduction in the point value of a parameter. In the context of this analysis, this means that scale of fluctuation computed with the SVF will always lead to a greater reduction in the point value of a parameter. It should be noted that both approaches to the determination of the scale of fluctuation in the context of the VRF will either lead to no reduction in the point variance, in which case the VRF is one, or lead to a reduction in the point variance, in which case the VRF will be less than one.

Since both approaches can either deliver no reduction or some reduction in the point variance, the decision as to the approach to adopt should be based on the restrictions/conditionalities associated with the use of each approach. In this case the SVF approach will be adopted for reasons discussed elsewhere in §4.2.6 particularly the number of data required for analysis by each approach: N/4 for the ACF and N/2 for the SWF, where N is the total number of data points. I think the higher the number of data that goes into an analysis the better.

This analysis is based on seven soundings and therefore is by no means exhaustive. Further studies involving many more soundings are required to come to firm conclusions on this subject.

6. PROPOSED FRAMEWORK AND ITS APPLICATION

6.1. INTRODUCTION

A framework is proposed in this section, which incorporates the spatial averaging effect of parameters based on the scale of fluctuation and variance reduction factor that are computed from widely-spaced irregular and non-continuous data. Also presented in this section are illustrations of the application of the framework to RBD.

6.2. PROPOSED FRAMEWORK

The deployment of RBD requires that geotechnical parameters are characterized and analyzed statistically. In order to characterize geotechnical parameters fully, knowledge of their mean, variance, and scale of fluctuation is required (Vanmarcke, 1977). While the mean and variance are simply-obtained statistics, obtaining the scale of fluctuation is somewhat complex. This is particularly true in the case of widely-spaced, non-continuous, irregular data which does not yield a well-defined profile. The determination of the scale of fluctuation requires a well-defined profile. Most published literature on scale of fluctuation is based on continuous data at close and regular intervals (field data from cone penetrometer soundings) which does not yield a well-defined profile.

A framework that incorporates the spatial averaging effect resulting from the scale of fluctuation determined from relatively closely-spaced data (relative to CPT data) into RBD is proposed herein. The steps involved in the framework are presented as follows:

- Determine the scale of fluctuation from irregularly-spaced data obtained from laboratory tests on samples obtained from the continuous Shelby tube sampling using the semivariogram function from geostatistics (Matheron, 1963). Geostatistical methods have been used extensively in other branches of civil engineering like environmental engineering, and hydraulics/hydrology. It should be noted that while this study is based on data from laboratory tests on samples obtained from the continuous Shelby tube sampling, the principles applied here are applicable to data from other kinds of test, for example the CPT where the sounding data is normally collected at close intervals.
- Use the scale of fluctuation to develop a plot of variance reduction factor against averaging length. This is done with a view to taking advantage of the spatial averaging effect in spatial variability analyses. This spatial averaging effect is the statistical scale effect whereby the variability of geotechnical parameters decreases as the size of the domain where it is defined increase. Hence, spatially averaged values of geotechnical parameters are usually smaller than their point values. The variance reduction factor, $\Gamma^2(L)$, is defined as the ratio of the point variance, σ_i^2 and the variance of the spatially averaged property, σ_L^2 (Vanmarcke, 1977).
- Obtain the variance reduction factor for the averaging distance, and then reducing the point variance of the parameters needed for design by the variance reduction factor.
- Update the variance of the geotechnical parameters to account for its spatial variation as stated above, and then using the updated variance in the limit state

function under consideration, alongside the associated mean value, and the values of any other probabilistic and, or deterministic parameters required for design.

The step-by-step procedure for this framework which incorporates the procedure for the determination of the scale of fluctuation using the semivariogram function (presented in Section 4) is presented below. A flowchart of the framework is presented in Figure 6.1.

1. Develop the profile of the parameter.
2. Validate the profile – check for outliers
3. Update the profile of the parameter.
4. Check the profile for the presence of any trend
    a. If a trend is present (dataset is nonstationary), make it stationary by removing a low-order polynomial trend, usually no higher than a quadratic, using the method of Ordinary Least Squares, OLS.
    b. Check stationarity by visual inspection of a scatter-plot of the data, and or tests like the Runs test, and the Kendal's τ test.
5. Compute and plot the experimental semivariogram using the lag as the distance (usually fixed) at which data is acquired. Use the original data in the computation if there is no trend present. Alternatively, if there is trend present use the residuals from the detrending process.
    a. For continuously acquired data at close, and regular intervals, use the Equation 25a shown below to compute the experimental variogram:

$$\gamma^{*}(h) = \frac{1}{2n(h)} \sum_{i=1}^{n(h)} [g(x_i) - g(x_i + h)]^2 \qquad (25a)$$

b. For non-continuously acquired data at wide, and irregular intervals, use the Equation 25b shown below to compute the experimental variogram:

$$\gamma^* \left(h \pm \Delta h\right) = \frac{1}{2n(h \pm \Delta h)} \sum_{i=1}^{n(h)} [g(x_i) - g(x_i + h)]^2 \qquad (25b)$$

6. Select a best-fit theoretical semivariogram model and fit it to the experimental semivariogram.
7. Determine the properties (the nugget effect, *Co*; the sill, *C* + *Co*; and the range of influence, *a*) of the fitted semivariogram model.
8. Select the relationship for the scale of fluctuation corresponding to the theoretical semivariogram model selected in (6). Some relationships are presented in Table 2.3.
9. Compute the scale of fluctuation using the relationship selected in (8).
10. Assuming hypothetical averaging lengths, compute variance reduction factor using the Equations 26a and 26b shown below.

$$\Gamma^2(L) = \left[\frac{\theta}{L}\left(1 - \frac{\theta}{4L}\right)\right]^{\frac{1}{2}} \qquad \text{for } L/\theta > 1/2 \qquad (26a)$$

$$\Gamma^2(L) = 1 \qquad \text{for } L/\theta \leq 1/2 \qquad (26b)$$

11. Develop relationships (in form of charts) between the variance reduction factor and averaging length.

12. For a known averaging length, deduce the variance reduction factor and then reduce the variance of the parameter by the variance reduction factor.
13. Use the reduced variance in the RBD computations.

6.3. EXAMPLES OF THE PROPOSED FRAMEWORK

The application of the framework is illustrated by the design of two foundation structures according to the First-Order, Second-Moment (FOSM):

- A shallow foundation to illustrate the effect of a small averaging length; and
- A deep foundation, say a pile of substantial length, to illustrate the effect of a large averaging length.

6.3.1. Illustration 1 – Shallow Footing. The effect of considering the spatial variability of geotechnical parameters in reliability-based design of a shallow foundation is illustrated with the following example.

In this example a 4’ x 4’ foundation is designed for a Factor of Safety (FS) of 4 using data from Warrensburg, Borehole 2 (W-BH2). Geotechnical design parameters including undrained shear strength S_u (UU) and effective friction angle, φ’ are presented in Tables 6.1 and 6.2 and Figure 6.2 while the schematic plot for the example is presented in Figure 6.3.

The First Order, Second Moment reliability design method (Duncan, 2000; Stephenson, 2009b) is used in this illustration.

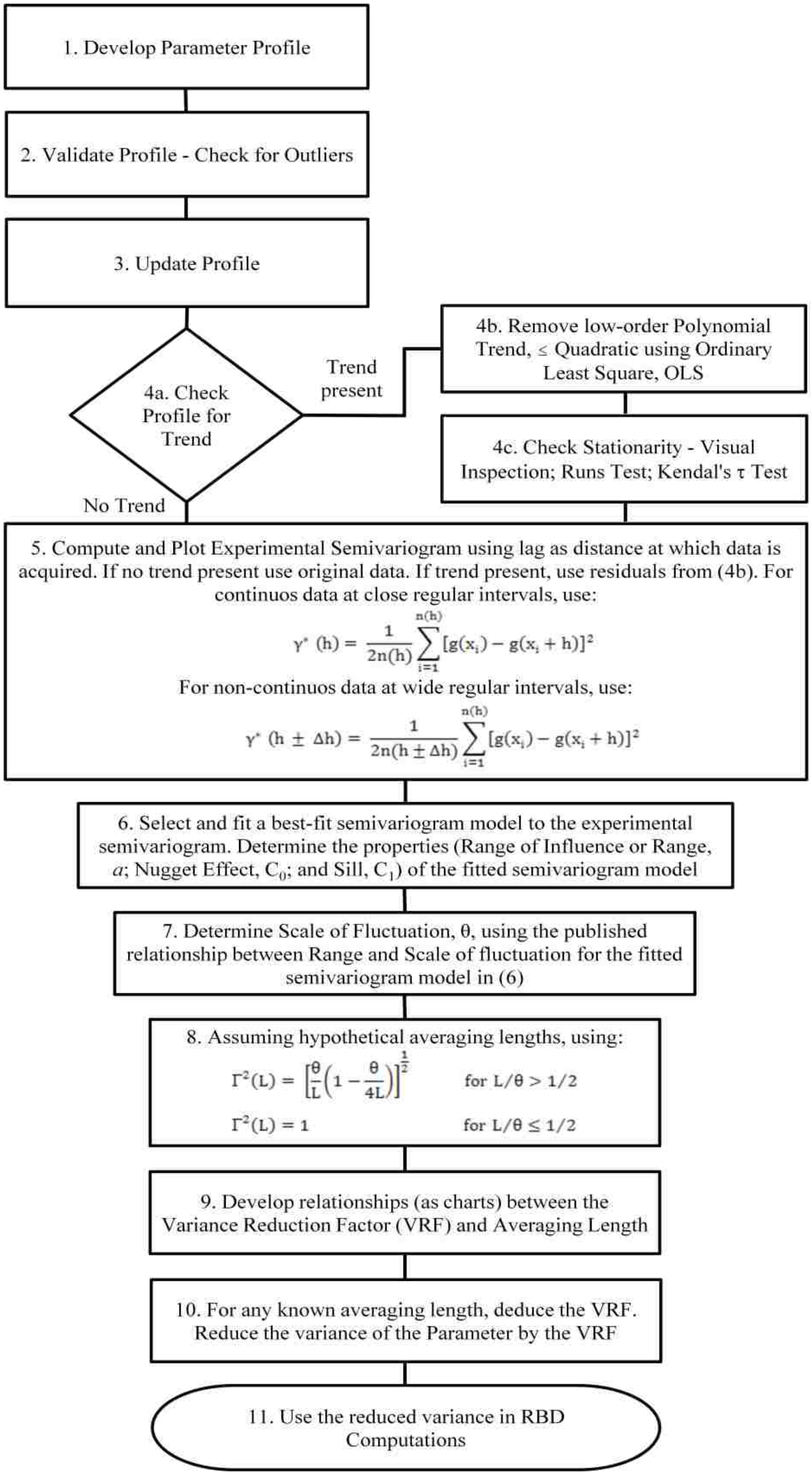


Figure 6.1: Flowchart of the Proposed Framework

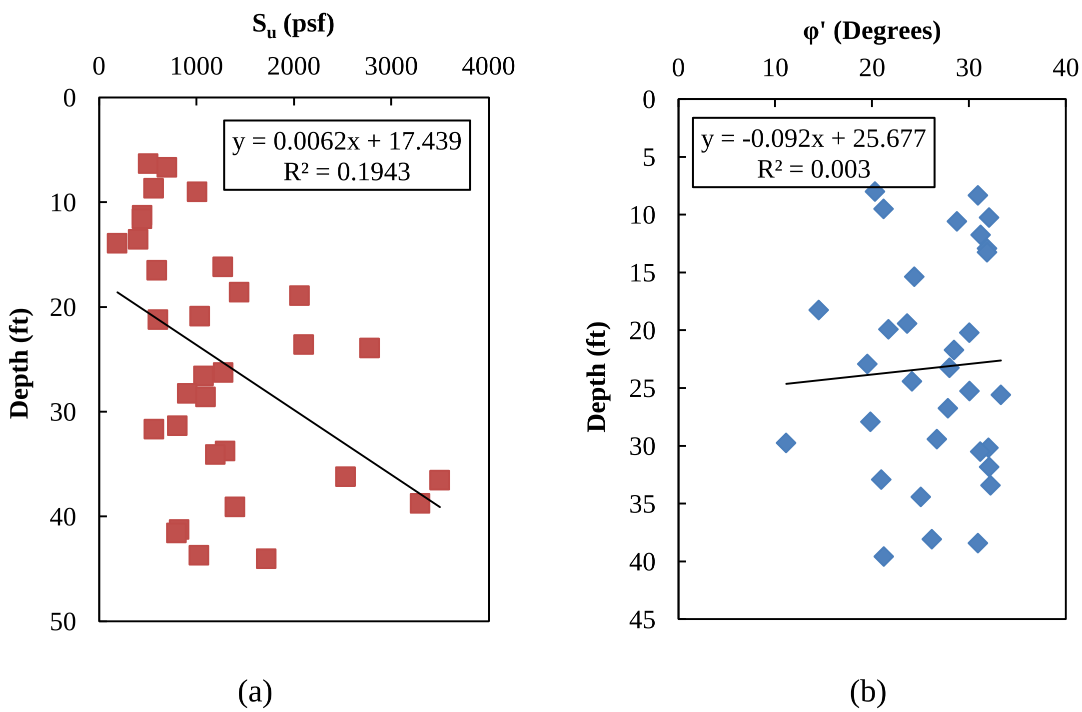


Figure 6.2: W-BH2 Profile for (a) Undrained Shear Strength, S_u (UU), (b) Effective Friction Angle, φ’

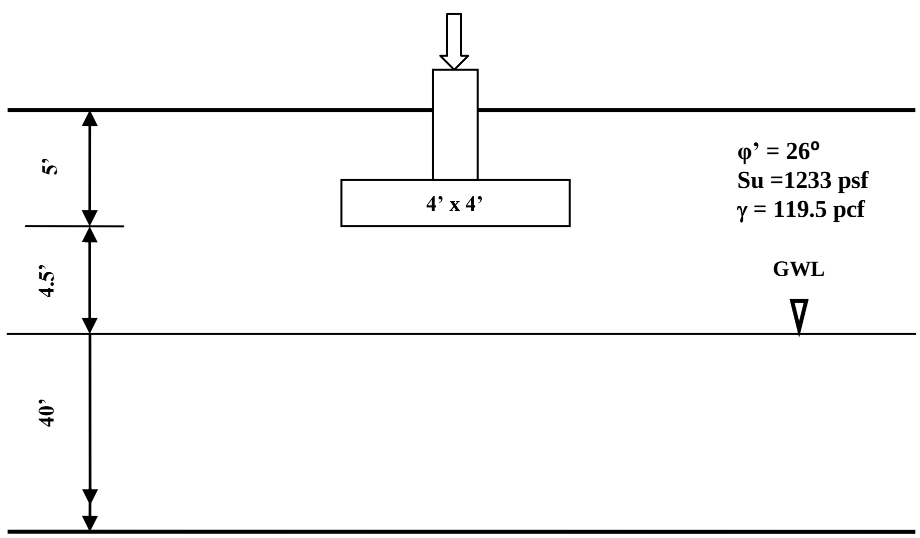


Figure 6.3: Schematic Plot for the Shallow Footing Design Illustration

Table 6.1: Undrained Shear Strength, S_u (UU) Data

W – BH 2							
Depth	S_u (UU)	Depth	S_u (UU)	Depth	S_u (UU)	Depth	S_u (UU)
6.3	506.0	16.2	1272.0	26.3	1276.0	26.3	1276.0
6.7	698.0	16.5	594.0	26.6	1074.0	26.6	1074.0
8.7	561.6	18.6	1440.0	28.3	907.2	28.3	907.2
9.0	1008.0	18.9	2059.2	28.6	1094.4	28.6	1094.4
11.2	444.0	20.9	1035.0	31.3	805.0	31.3	805.0
11.6	442.0	21.2	606.0	31.7	565.0	31.7	565.0
13.5	403.2	23.6	2102.4	33.8	1296.0	33.8	1296.0
13.9	187.2	23.9	2779.2	34.1	1195.2	34.1	1195.2
Note: Depth = feet; S_u = Undrained Shear Strength (psf)							

Table 6.2: Friction Angle, φ' Data

W – BH 2							
Depth	φ'	Depth	φ'	Depth	φ'	Depth	φ'
8.0	20.3	15.4	24.4	24.4	24.1	30.5	31.2
8.3	30.9	18.3	14.5	25.6	33.3	31.8	32.1
9.5	21.2	19.4	23.6	25.3	30.1	32.9	21.0
10.3	32.1	19.9	21.7	26.8	27.8	33.4	32.2
10.6	28.8	20.2	30.1	29.8	11.1	34.4	25.0
11.8	31.2	21.7	28.5	29.4	26.7	38.1	26.2
12.9	31.9	22.9	19.5	27.9	19.8	38.4	30.9
13.3	31.9	23.3	28	30.2	32.0	39.6	21.2
Note: Depth = feet; φ' = effective friction angle (degree)							

From the plots of the semivariogram for the S_u (UU) and φ' profiles presented in Figures 6.4 and 6.5, the range of influence (a) and the scale of fluctuation (θ) is 3.9 ft, 2.9 ft and 2.4 ft, 1.8 ft, respectively. With the scale of fluctuation known, the plots of variance reduction factor against averaging length for the S_u (UU) and φ' profiles were developed by assuming hypothetical averaging lengths and substituting the scale of fluctuation, θ, of the S_u (UU) and φ' profiles into the Equation 26. The plots of the variance reduction factor against averaging length for S_u (UU) and φ' are presented in

Figure 6.6 and 6.7, respectively. The variance reduction factor of S_u (UU) and φ' for 13 feet averaging length is about 46% and 37%, respectively.

The bearing capacities are computed using the generalized bearing capacity equation (Das, 2007):

$$q_u = c'N_C F_{cs} F_{cd} F_{ci} + qN_q F_{qs} F_{qd} F_{qi} + 0.5\gamma B N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i} \tag{27}$$

where c' = cohesion, q = effective stress at the level of the bottom of the foundation, γ = unit weight of soil, *B* = width of foundation (or diameter for a circular footing), F_{cs}, F_{qs}, $F_{\gamma s}$ = shape factors, F_{cd}, F_{qd}, $F_{\gamma d}$ = depth factors, F_{ci}, F_{qi}, $F_{\gamma i}$ = load inclination factors, and N_c, N_q, N_γ = bearing capacity factors.

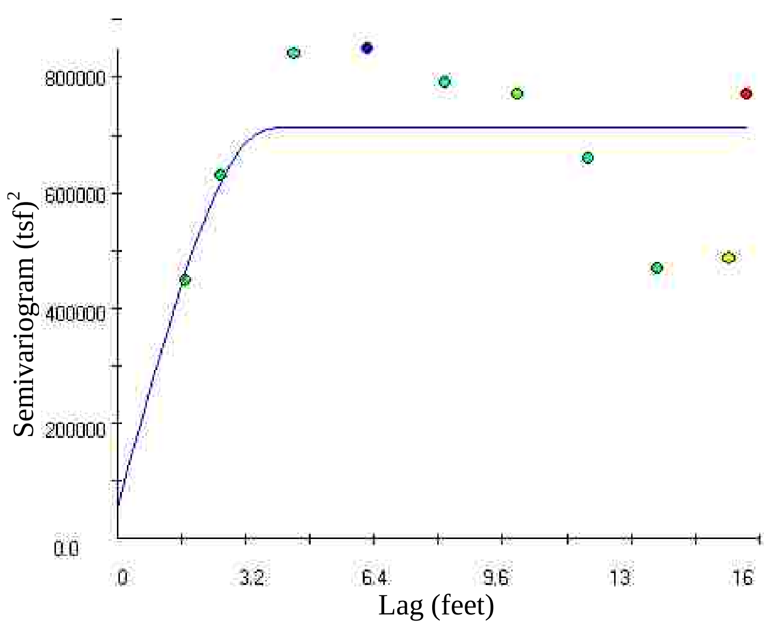


Figure 6.4: Semivariogram for Su (UU) – Type: Spherical; a = 3.9 ft; θ = 2.9 ft

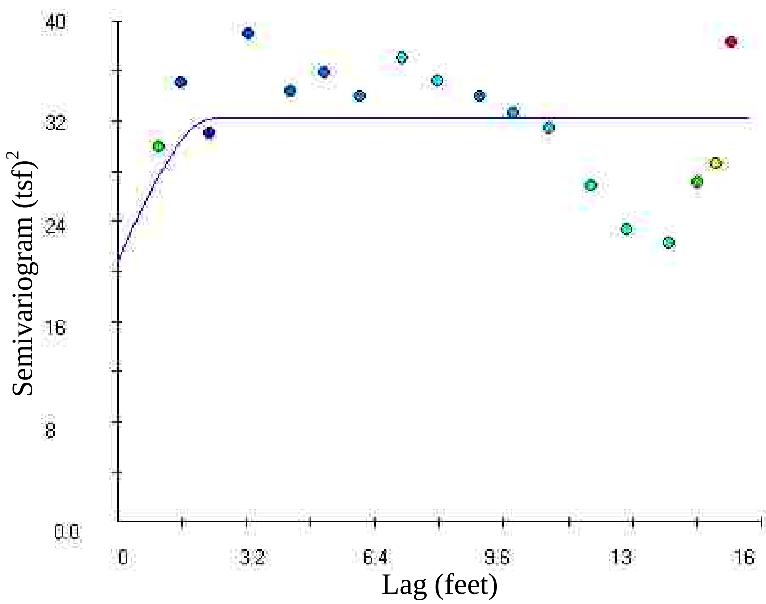


Figure 6.5: Semivariogram for φ’ – Type: Spherical; a = 2.4 ft; θ = 1.8 ft

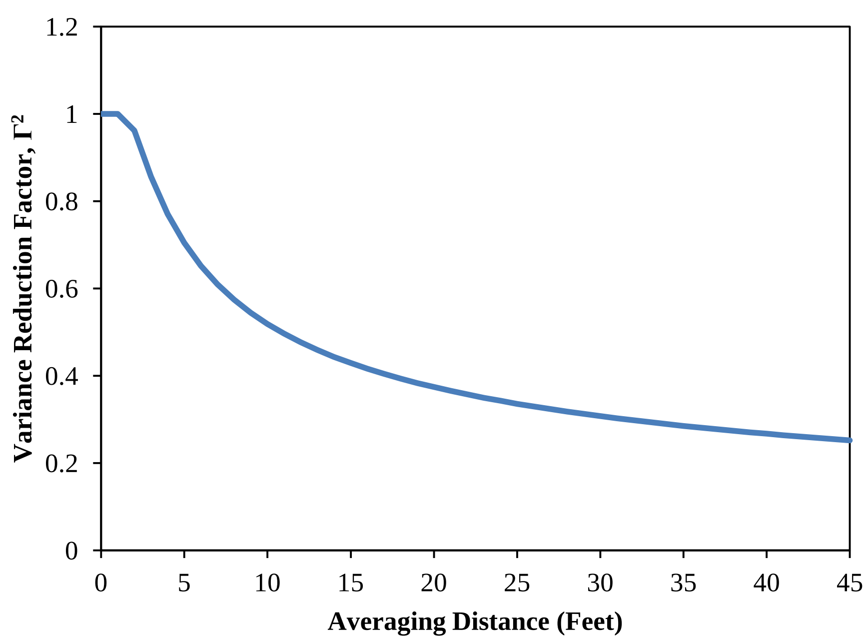


Figure 6.6: Variance Reduction Factor for Undrained Shear Strength, S_u (UU)

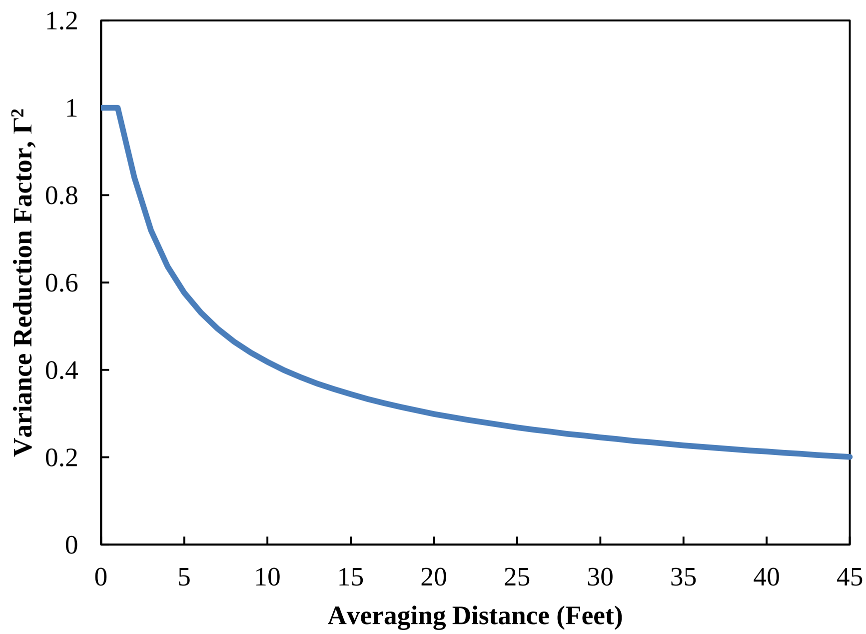


Figure 6.7: Variance Reduction Factor for Effective Friction Angle, φ’

The parameters considered are S_u (UU) and φ’ for total stress and effective stress analyses, respectively. The unit weight was not considered because the variability is usually low (COV = 4.5% in this case). The second moment statistics and the spatially-averaged values of S_u (UU) and φ’ are presented on Table 6.3.

Table 6.3: Second Moment Statistics and Spatially-Averaged Values (Footing)

Statistic	φ’ (deg)	S_u (UU) (psf)	Statistic	φ’ (deg)	S_u (UU) (psf)
Second Moment			Spatially Averaged		
Mean	26	1233	Mean	26	1233
Std. deviation	8	835	Std. deviation	5	528
Variance	63	696632	Variance	25	278653

For the total stress computations, φ' was zero, and the MLV, MLV plus 1 standard deviation (Stdev), and MLV minus 1 Stdev values of S_u, respectively were used to compute q_u. For the effective stress computations, S_u was zero, and the MLV, MLV plus 1 standard deviation (Stdev), and MLV minus 1 Stdev values of φ', respectively were used to compute q_u. The MLV, MLV plus 1 standard deviation (Stdev), and MLV minus 1 Stdev values for the parameters are presented in Tables 6.4a and 6.5b for the second moment and the spatially averaged values, respectively. Tables 6.6a and 6.7b present the ultimate bearing capacities for the MLV, MLV plus 1 standard deviation (Stdev), and MLV minus 1 Stdev, and the results of the reliability analyses for total stress and effective stress analyses, respectively.

Table 6.4: Data for Reliability Computations (Footing) – Second Moment Values

φ' (degree)			S_u (UU) (psf)		
[-1]Stdev	MLV	[+1]Stdev	[-1]Stdev	MLV	[+1]Stdev
18	26	34	398	1233	2067

Table 6.5: Data for Reliability Computations (Footing) – Spatially-Averaged Values

φ' (degree)			S_u (UU) (psf)		
[-1]Stdev	MLV	[+1]Stdev	[-1]Stdev	MLV	[+1]Stdev
21	26	31	666	1233	1799

For the reliability assessment, the ultimate allowable bearing capacity for the MLV, $Q_{all(MLV)}$ is assumed to be the applied stress, Q_{app}. The values in Tables 6.6 and 6.7 were computed using the following formulas:

- #1 - 4: computed using the generalized bearing capacity equation
- #5: $FS = Q_{ult} / Q_{app}$
- #6: $\Delta FS = FS_{max} - FS_{min}$
- #7: $\sigma_{FS} = ((\Delta FS / 2)^2)^{0.5}$; Standard deviation of FS
- #8: $v_{FS} = (\sigma_{FS} / FS_{MLV})$; Coefficient of variation of FS
- P_f : obtained from standard lognormal table [using FS_{MLV} and V_{FS} (%)]

Table 6.6: Reliability Computations (Footing) – Total Stress Analysis

#	Property	MLV	[+1]Stdev	[-1]Stdev	MLV	[+1]Stdev	[-1]Stdev
		S_u – Second Moment			S_u – Spatially-Averaged		
1	q_{ult} (psf)	9506.43	15936.57	3068.58	9506.43	13870.29	5134.86
2	q_{all} (psf)	2376.61	3984.14	767.15	2376.61	3467.57	1283.72
3	Q_{ult} (lbs)	152102.9	254985.1	49097.28	152102.9	221924.6	82157.76
4	Q_{all} (lbs)	38025.72	63746.28	12274.32	38025.72	55481.16	20539.44
5	FS	4	6.7	1.3	4	5.8	2.2
	Summary						
6	ΔFS	5.4			3.7		
7	σ_{FS}	2.7072			1.8378		
8	v_{FS}	0.6768	67.7%	P_f = 2.56%	0.4594	45.9%	P_f = 0.16%

Using the values of FS_{MLV} (= 4 in this case) and the values v_{FS}, assuming a lognormal distribution, the values of the probability of failure, P_f were determined. The values of P_f are presented on Tables 6.6 and 6.7.

From Table 6.6, it can be seen that taking the spatial variability into consideration in the total stress analysis resulted in a reduction of P_f. The reduction was from 2.56% to 0.16% (about one order of magnitude). Table 6.7 shows that no change in P_f for the effective stress analysis. Both P_f due to second moment and spatially-averaged values were zero.

Table 6.7: Reliability Computations (Footing) – Effective Stress Analysis

#	Property	MLV	[+1]Stdev	[-1]Stdev	MLV	[+1]Stdev	[-1]Stdev
		φ' – Second Moment			φ' – Spatially-Averaged		
1	q_{ult} (psf)	900.66	999.0	832.96	900.66	957.41	855.67
2	q_{all} (psf)	225.17	249.75	208.24	225.17	239.35	213.92
3	Q_{ult} (lbs)	14410.56	15983.93	13327.44	14410.56	15318.58	13690.74
4	Q_{all} (lbs)	3602.64	3995.98	3331.86	3602.64	3829.64	3422.68
5	FS	4	4.4	3.7	4	4.3	3.8
Summary							
6	ΔFS	0.7			0.5		
7	σ_{FS}	0.3687			0.2259		
8	v_{FS}	0.09218	9.2%	P_f = 0%	0.05648	5.6%	P_f = 0%

6.3.2. Illustration 2 – Deep Foundation. The effect of considering the spatial variability of geotechnical parameters in reliability-based design of a deep foundation is illustrated with the following example.

In this example a 40 feet long, 1.5 feet diameter, pile is designed for a Factor of Safety (FS) of 3 using data from Warrensburg, Borehole 2 (W-BH2). The Warrensburg, Borehole 2 (W-BH2) data for S_u (UU) and friction angle, φ' have been presented previously in Tables 6.1 and 6.2 and Figure 6.2. The schematic for this illustration is presented in Figure 6.8.

Like in the illustration for the shallow foundation, the First Order, Second Moment reliability design method (Duncan, 2000; Stephenson, 2009b) is used in this illustration.

From the plots of the semivariogram for the S_u (UU) and φ profiles presented in Figures 6.4 and 6.5, the range of influence and the scale of fluctuation is 3.9 ft, 2.9 ft and 2.4 ft, 1.83 ft, respectively. With the scale of fluctuation known, the plots of variance reduction factor against averaging length for the S_u (UU) and φ' profiles were developed by assuming hypothetical averaging lengths and substituting the scale of fluctuation, θ, of the S_u (UU) and φ' profiles into the Equation 26. The plots of the variance reduction factor against averaging length for S_u (UU) and φ' were presented in Figure 6.6 and 6.7, respectively. The averaging length for the skin friction is 40 feet while the averaging length for end bearing is three feet. The variance reduction factor of S_u (UU) and φ' for averaging lengths of three feet and 40 feet, respectively is about 21% and 27%.

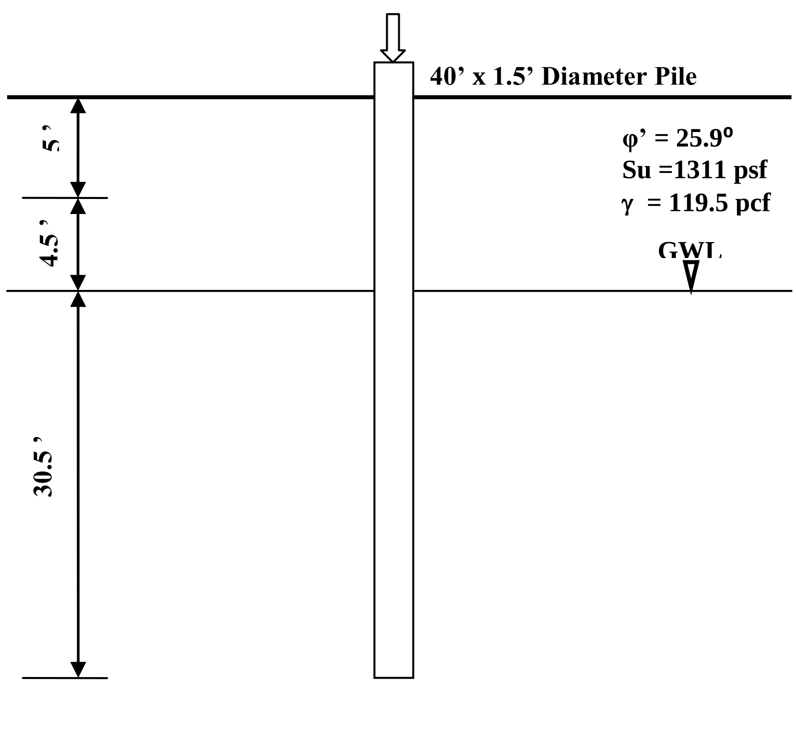


Figure 6.8: Schematic Plot for the Deep Foundation Design Illustration

The ultimate load capacity, Q_{ult} of the pile is determined using the static pile load capacity equation (Das 2007; Budhu 2007):

$$Q_{ult} = Q_f + Q_b + W_p \tag{28}$$

Where: Q_f = skin friction; Q_b = end bearing; and W_p = weight of pile.

The parameters considered are S_u (UU) and φ'. The unit weight was not considered because the variability is usually low (COV = 4.5% in this case). The second moment statistics and the spatially averaged values of S_u and φ' are presented on Table 6.8.

Table 6.8: Second Moment Statistics and Spatially-Averaged Values (Pile)

Statistic	φ' (deg)	S_u (UU) (psf)	φ'^{1} (deg)	φ'^{2} (deg)	S_u (UU)(psf)
Second Moment			Spatially Averaged		
Mean	25.9	1311	25.9	25.9	1311
Std. deviation	57	863	23.1	6.7	233
Variance	32.1	745014	4.1	1.2	201153
Notes:1 = Tip resistance; 2 = Side friction					

In this illustration, the reliability of the ultimate load capacity for the short-term and the long-term loading conditions is determined using the α method and β methods (Das, 2007; Budhu, 2007), respectively.

In the computations, as only one parameter is required in the equation for end bearing and side friction, respectively, the parameter values were considered simultaneously; i.e. mean values (or most likely value, MLV) for φ' and MLV for Su (UU) considered for end bearing and side friction simultaneously. The MLV, MLV plus 1 standard deviation (Stdev), and MLV minus 1 Stdev values for the parameters are presented in Tables 6.9 and 6.12 for the second moment and the spatially averaged values, respectively. The summary of the result of the ultimate pile load capacity computations are presented in Tables 6.10 and 6.13 for the second moment and the spatially averaged values, respectively. The reliability analyses of the ultimate pile load capacity for the MLV, MLV plus 1 standard deviation (Stdev), and MLV minus 1 Stdev are presented in Tables 6.11 and 6.14 for the second moment and the spatially averaged values, respectively.

Table 6.9: Data for Reliability Computations (Pile) – Second Moment Values

φ' (deg)			S_u (UU) (psf)		
[-1]Stdev	MLV	[+1]Stdev	[-1]Stdev	MLV	[+1]Stdev
20.2	25.9	31.6	446	1311	2173

Table 6.10: Pile Capacity Computations – End Bearing and Side Friction, Second Moment Values

Parameter	Total Stress Analysis			Effective Stress Analysis		
	[-1]Stdev	MLV	[+1]Stdev	[-1]Stdev	MLV	[+1]Stdev
Q_b (lbs)	7093.3	20850.6	34560.1	36098.8	74029.2	151814.6
Q_f (lbs)	71425.0	112578.0	193495.4	76070.5	86362.7	92480.2
Q_{ult} (lbs)	78518.4	133428.6	228055.5	112169.4	160392.0	244294.8

For the reliability assessment, the ultimate allowable bearing capacity for the MLV, $Q_{all(MLV)}$ is assumed to be the applied stress, Q_{app}. The values in Tables 6.6c and 6.7c were computed using the following formulas:

- #1: computed using static pile load capacity equation
- #2: $Q_{all} = Q_{ult} / FS$
- #3: $FS = Q_{ult} / Q_{app}$
- #4: $\Delta FS = FS_{max} - FS_{min}$
- #5: $\sigma_{FS} = ((\Delta FS / 2)^2)^{0.5}$; Standard deviation of FS
- #6: $v_{FS} = (\sigma_{FS} / FS_{MLV})$; Coefficient of variation of FS
- P_f : obtained from standard lognormal table [using FS_{MLV} and V_{FS} (%)]

Table 6.11: Reliability Computations (Pile) – Second Moment Values

#	Property	MLV	[+1]Stdev	[-1]Stdev	MLV	[+1]Stdev	[-1]Stdev
		S_u (UU) - Total Stress Analysis			φ' - Effective Stress Analysis		
1	Q_{ult} (lbs)	133428.6	228055.5	78518.4	160392.0	244294.8	112169.4
2	Q_{all} (lbs)	44476.2	76018.5	26172.8	53464.0	81431.6	37389.8
3	FS	3.0	5.1	1.8	3.0	4.6	2.1
Summary							
4	ΔFS	3.4			2.5		
5	σ_{FS}	1.6811			1.2356		
6	v_{FS}	0.5604	56.04%	P_f = 3.282%	0.4119	41.19%	P_f = 0.498%

Using the values of FS_{MLV} (3 in this case) and the values v_{FS}, assuming a lognormal distribution, the values of the probability of failure, P_f were determined. The values of P_f are presented in Tables 6.11 and 6.14.

Table 6.12: Data for Reliability Computations (Pile) – Spatially-Averaged Values

φ' (degree) - Tip			φ' (degree) - Side			Su (UU) (psf)		
[-1] Stdev	MLV	[+1] Stdev	[-1] Stdev	MLV	[+1] Stdev	[-1] Stdev	MLV	[+1] Stdev
21.8	25.9	30	24.7	25.9	27.1	1077	1311	1543

Table 6.13: Pile Capacity Computations – End Bearing and Side Friction, Spatially-Averaged Values

Parameter	Total Stress Analysis			Effective Stress Analysis		
	[-1]Stdev	MLV	[+1]Stdev	[-1]Stdev	MLV	[+1]Stdev
Q_b (lbs)	17121.0	20850.6	24534.0	44187.5	74029.2	123438.3
Q_f (lbs)	121424.8	112578.0	151085.6	84543.0	86362.7	87947.0
Q_{ult} (lbs)	138545.8	133428.6	175619.6	128730.5	160392.0	211385.3

Table 6.14: Reliability Computations (Pile) – Spatially-Averaged Values

#	Property	MLV	[+1]Stdev	[-1]Stdev	MLV	[+1]Stdev	[-1]Stdev
		S_u (UU) - Total Stress Analysis			φ' - Effective Stress Analysis		
1	Q_{ult} (lbs)	133428.6	175619.6	138545.8	160392.0	211385.3	128730.5
2	Q_{all} (lbs)	44476.2	58539.9	46181.9	53464.0	70461.8	42910.2
3	FS	3.0	3.9	3.1	3.0	4.0	2.4
Summary							
4	ΔFS	0.9			1.5		
5	σ_{FS}	0.4743			0.7730		
6	v_{FS}	0.1581	15.81%	P_f = 0%	0.2577	25.77%	P_f = 0.001%

From Tables 6.11 and 6.14, it can be seen that taking the spatial variability of S_u (UU) in the total stress analysis (short-term loading condition) and the spatial variability of φ' in the effective stress analysis (long-term loading condition) resulted in a reductions in the P_f. For total stress analysis [S_u (UU)], the reduction was from 3.282% to 0% (about one order of magnitude) while for effective stress analysis (φ') the reduction was from 0.498% to 0.001% (about two orders of magnitude). The reduction for total stress analysis was actually from 3.282% to 2.366E-12%.

The examples above are for the case where there are sufficient data for analyses. In the case where there are insufficient data for analysis, a combination of approximate methods, expert knowledge, and engineering judgment will have to be applied to determine the values of the parameters to be used in design.

The above illustrations demonstrate that applying the proposed framework in design leads to a decrease in the variance of the design parameters due to the spatial averaging effect. Spatial averaging in both illustrations resulted in the reduction of the probability of failure. Such reductions in the probability of failure, under certain circumstances, translate to economic savings. This is particularly true for the case where due to a large reduction in the probability of failure, the size (dimensions) of a structure can be reduced.

It is recommended that public transportation agencies, such as MoDOT adopt the framework for their designs. To deploy the framework, MoDOT will have to adopt the continuous Shelby tube sampling method of soil exploration. With continuous Shelby tube sampling, tests can be assigned at about one foot interval or less and hence it is possible to obtain sufficient data for semivariogram analysis. A minimum of about 25 data points will be sufficient for semivariogram analysis. Following from the semivariogram analysis, the scale of fluctuation and the variance reduction factor can be determined. The parameter variance for design is reduced accordingly variance reduction factor.

7. CONCLUSION AND RECOMMENDATION

7.1. INTRODUCTION

Based on data obtained from laboratory tests on samples obtained from the continuous Shelby tube sampling method and field (CPTu) investigation program in Missouri, the study reported herein is aimed at increasing the use of RBD among geotechnical engineers. The main objectives of this study are as follows:

- Characterize fine-grained soils in Missouri for reliability analyses;
- Develop an RBD framework incorporating the spatial variability of geotechnical parameters based on widely-spaced, non-continuous, irregular data obtained from laboratory tests on specimens obtained by the continuous sampling borehole exploration method; and
- Demonstrate the application of the proposed framework.

To achieve the objectives of this study a coordinated and carefully executed research program incorporating desk study, laboratory and field investigations, data analyses, and the application of the results of data analysis to RBD work was undertaken. Also a framework for the application of the results of the data to RBD was proposed and illustrations of the use of this framework were also presented. A summary of major findings and major conclusions of the research study and recommendations as to future research needs and directions are presented in this section.

7.2. CONCLUSIONS

The following conclusions are drawn on the basis of the analyses conducted in this study. These conclusions are true for the data used in the various analyses conducted in this study.

- The second moment statistics and the probability distribution of both laboratory-determined and measured CPTu parameters were found to be dependent on both in-situ state and soil classification type. The change in the second moment statistics and the probability distribution as result of grouping data into in-situ state and, or, soil classification type followed no discernable pattern.
- For the research data, correlations between Su/σ' and PI were found to be largely non-linear (second degree polynomials) and not linear as found in most published empirical relationships. There is low degree correlation between Su/σ' and PI. The coefficients of determination for the empirical relationships developed between overburden stress-normalized undrained shear stress (UU) and PI were generally low (below 0.70) when considered in terms of location (Warrensburg = 0.16; St Charles = 0.47; New Florence = 0.68; Pemiscot = 0.19) and also when considered in terms of soil classification type (CL = 0.48; CH = 0.37; ML = 0.39).
- For the empirical correlations with strength parameters, published empirical relationships between overburden stress- and preconsolidation stress-normalized undrained shear stress (UU and CU) and Atterberg limits and, or, other easily-obtained geotechnical parameters were found not valid for the data used in this study.

- For the empirical correlations with consolidation parameters, published empirical relationships between compression index and Atterberg limits and, or, other easily-obtained geotechnical parameters were found to be valid for the data used in this study. The empirical correlations between recompression index and Atterberg limits and, or, other easily-obtained geotechnical parameters were found not valid for the data used in this study.
- Scale of fluctuation of geotechnical parameters is dataset-specific. When determined using the semivariogram function it depends also on the type of trend removed from the data in the detrending process, and the type of model employed in the variogram analysis.
- For laboratory test data, which compared to CPTu data are relatively very widely-spaced and obtained at irregular space intervals, the scale of fluctuation is best obtained using the semivariogram function.
- Preliminary analyses indicate that there are differences in the scale of fluctuation determined using laboratory test data and CPTu data. These differences follow no particular pattern but may be dependent on the type of semivariogram model employed in the analysis. For instance for CPTu data, semivariogram models which reach their asymptotically like the exponential model gave a high range of influence while models which reach their sill at the range of influence like the spherical model gave a lower range of influence.
- Preliminary analyses, using CPTu data, indicate that the average scale of fluctuation determined using the semivariogram function is higher than that

determined using the autocorrelation function. The magnitude of the difference is in the range of 4 to 400%.

- For CPTu data, taking the number of data used in analysis, the scale of fluctuation is best obtained using the semivariogram function. The semivariogram function uses N/2 while the autocorrelation function uses N/4 for analyses; where N is the total number of data points.
- The framework for RBD proposed and the examples of its application presented are based on the case where there are sufficient data. In cases where data is insufficient, the combined application of approximate methods, local knowledge, and expert knowledge is warranted.
- Incorporating the spatial averaging effect in RBD leads to either no change or a reduction in the variance but never an increase the variance.
- Incorporating the spatial averaging effect in RBD leads to a reduction in the probability of failure of a structure. Reductions in the probability of failure of a structure could lead to a reduction in its dimensions and hence lead to cost savings. In the examples presented in this study, reductions in probability of failure of up to two orders of magnitude were reported.

7.3. RECOMMENDATIONS

This study has demonstrated the statistical characterization of geotechnical parameters and their application to RBD. However, there are some observations arising from the study that requires further investigation. These observations are stated following.

- The determination of the both the second moment statistics and probability distribution of geotechnical parameters in terms of both their in-situ state and soil classification types.
- The development of correlation models between overburden stress-normalized undrained shear stress (UU and CU) and other easily-obtained geotechnical parameters that includes not only the Atterberg limits but the other parameters that influence the undrained shear strength like preconsolidation pressure, overconsolidation ratio, void ratio, and etc.
- Further studies, involving more boreholes and more semivariogram models, on the relationship between the range of influence from CPTu data and the range of influence from other more prevalent sampling and testing methods as amended.
- Further studies, involving more CPTu soundings, on the relationship between the scales of fluctuation computed using the autocorrelation function and the semivariogram function.
- MoDOT adopts the continuous sampling method of soil exploration to provide sufficient data points (25 minimum) to take advantage of the reduced parameter variance of the proposed framework. The reduced parameter variance leads to optimal/economical designs.

APPENDIX

FIELD EXPLORATION AND LABORATORY TESTING, CPTu SOFTWARE CORRELATIONS, SECOND MOMENT STATISTICS, PEARSON'S SPACE PLOTS, DISTRIBUTION TYPE & CORRELATION MATRIX ON CD-ROM

1. INTRODUCTION

Included with this Dissertation is a CD-ROM which contains the eight Appendices (Appendices A to H) referred to in this study. Appendices A and B are Microsoft Word 2007 document files while Appendices C to H are Microsoft Excel 2007 files. An outline of the contents of the CD-ROM is as follows.

2. CONTENTS

APPENDIX A: FIELD EXPLORATION AND LABORATORY TESTING

APPENDIX B: CPeT-IT

APPENDIX C: SECOND MOMENT STATISTICS – LABORATORY TEST DATA

APPENDIX D: SECOND MOMENT STATISTICS - CPTu DATA

APPENDIX E: PEARSON'S SPACE

APPENDIX F: DISTRIBUTION TYPE - LABORATORY TEST DATA

APPENDIX G: DISTRIBUTION TYPE - CPTu DATA

APPENDIX H: CORRELATION MATRIX

REFERENCES

Anderson, O. D. (1976). Time Series Analysis and Forecasting: The Box-Jenkins Approach. London: Butterworths.

Ang, A. H-S., and Tang, W. H. (2007). Probability Concepts in Engineering. Hoboken, NJ: Wiley.

Arman, A and McManis, K. L. (1976). "Effects of Storage and Extrusion on Sample Properties," *ASTM STP 599*, pp. 66-87. Philadelphia, PA: American Society for Testing and Materials.

Asaoka, A., and A-Grivas, D. (1982). "Spatial Variability of the Undrained Strength of Clays," *Journal of Geotechnical Engineering (ASCE)*, Vol. 108, No. GT5, pp. 743-756. New York: ASCE.

ASTM D 2216 – 05. (2005). Standard Test Methods for Laboratory Determination of Water (Moisture) Content of Soil and Rock by Mass. West Conshohocken, PA: ASTM International.

ASTM D 2435 – 04. (2004). Standard Test Method for One-Dimensional Consolidation Properties of Soils. West Conshohocken, PA: ASTM International.

ASTM D 2850 – 03a. (2007). Standard Test Method for Unconsolidated-Undrained Triaxial Compression test on Cohesive Soils. West Conshohocken, PA: ASTM International.

ASTM D 422 – 63. (2007). Standard Test Method for Particle-Size Analysis of Soils. West Conshohocken, PA: ASTM International.

ASTM D 4318 – 05. (2005). Standard Test Methods for Liquid Limit, Plastic Limit, and Plasticity Index of Soils. West Conshohocken, PA: ASTM International.

ASTM D 4767 – 04. (2004). Standard Test Method for Consolidated Undrained Triaxial Compression test for Cohesive Soils. West Conshohocken, PA: ASTM International.

ASTM D4186 – 06. (2006). Standard Test Method for One-Dimensional Consolidation Properties of Saturated Cohesive Soils Using Controlled-Strain Loading. West Conshohocken, PA: ASTM International.

Azzoz, A. S., Krizek, R. J., and Corotis, R. B. (1976). "Regression Analysis of Soil Compressibility," *Soils and Foundations*, 16(2), pp. 19-29. Tokyo: Japanese Society of Soil Mechanics and Foundation Engineering.

Baecher, G. B. (1982). “Simplified Geotechnical Data Analysis.” In *Reliability Theory and Its Applications in Structural and Soil Engineering*, Proceedings of the NATO Advanced Study Institute Conference on Reliability Theory and Its Application in Structural and Soil Mechanics, P. Thoft-Christensen (ed.), Bornholm, Aug. 31-Sept. 9, 1982). Dordrecht: Reidel Publishing.

Baecher, G. B. (1984). “Geostatistics, Reliability, and Risk Assessment in Geotechnical Engineering.” In *Geostatistics for Natural Resources Characterization* (NATO ASI Series C, Volume 122), Part 2, G. Verly, M. David, A. G. Journel, and A. Marechal (eds.), pp. 731-744. Dordrecht, Holland: J. Reidel Publishing.

Baecher, G. B. (1986). “Geotechnical Error Analysis,” *Transportation Research Record*, No. 1105, pp. 23-31. Washington, DC: Transportation Research Board/National Research Council.

Baecher, G. B., and Christian, J. T. (2003). Reliability and Statistics in Geotechnical Engineering. Chichester, UK: Wiley.

Bjerrum, L., and Simons, N. E. (1960). “Comparison of Shear Strength Characteristics of Normally Consolidated Clays,” *1st PSC, ASCE*, pp. 711-726. New York: ASCE.

Bowles, J. E. (1997). Foundation Analysis and Design. New York: McGraw-Hill.

Box, G. E. P., and Jenkins, G. M. (1970). Time Series Analysis: Forecasting and Control. San Francisco: Holden-Day.

Box, G. E. P., Jenkins, G. M., and Reinsel, G. C. (1994). Time Series Analysis: Forecasting and Control, 3e. Upper Saddle River, NJ: Prentice Hall.

Budhu, M. (2008). Foundations and Earth Retaining Structures. New York: Wiley.

Cafaro, F., and Cherubini, C. (2002). “Large Sample Spacing in Evaluation of Vertical Strength Variability of Clayey Soil,” *Journal of Geotechnical and Geoenvironmental Engineering (ASCE)*, Vol. 128, No. 7, pp. 558-568. Reston, VA: ASCE.

Chandler, R. J. (1988). “The In-Situ Measurement of the Undrained Shear Strength of Clays using the Field Vane.” In *Vane Shear Strength Testing in Soils: Field and Laboratory Studies (STP 1014)*, pp.13-44. Philadelphia: ASTM.

Cherubini, C., and Giasi, C. I. (1993). “The Coefficients of Variation of Some Geotechnical Parameters.” In *Proceedings of the Conference on Probabilistic Methods in Geotechnical Engineering*, K. S. Li and S. C. Lo (eds.), Canberra, Australia, February 10-12, pp.179-183. Rotterdam: AA Balkema.

Cherubini, C., Vessia, G., and Pula, W. (2007). "Statistical Soil Characterization of Italian Site for Reliability Analyses." In *Characterization and Engineering Properties of Natural Soils*, T. S. Tan, K. K. Phoon, D. W. Hight, and S. Leroueil (eds.), pp.2681-2706. London: Taylor and Francis.

Chiasson, P., Lafleur, J., Soulie, M., and Law, K. T. (1995). "Characterizing Spatial Variability of a Clay Soil by Geostatistics," *Canadian Geotechnical Journal*, 32: 1-10.

Clark, I. (1979). Practical Geostatistics. London: Applied Science.

Clark, I. and Harper, W. V. (2000). Practical Geostatistics 2000. Alloa, Scotland: Geostokos.

Corotis, R. B., Azzouz, A. S., and Krizek, R. J. (1975). "Statistical Evaluation of Soil Index Properties and Constrained Modulus." In *Proceedings of the 2nd International Conference on Applications of Statistics and Probability to Soil and Structural Engineering*, E. Schiltze (ed.), Aachen, September 15-18, 1975, pp. 273-294. Essen, Germany: Deutsche Gesellschaft fuer Erd-und Grundbau.

Cressie, N. A. C. (1993). Statistics for Spatial Data. New York: Wiley.

Das, B. M. (2007). Principles of Foundation Engineering, 6e. Toronto: Nelson.

Das, B. M. (2010). Principles of Geotechnical Engineering, 7e, SI edition. Stamford, Connecticut: Cengage.

Davis, J. C. (1986). Statictics and Data Analysis in Geology, 2e. New York: Wiley.

DeGroot, D. J. (1996). "Analysing Spatial Variability of In Situ Soil Properties." In *Uncertainty in the Geological Environment: From Theory to Practice*, Geotechnical Special Publication No. 58, C. D. Shackleford, P. P. Nelson, and M. J. S. Roth (eds.), pp 210 - 238. New York: ASCE.

DeGroot, D. J., and Baecher, G. B. (1993). "Estimating Autocovariance of In-Situ Soil Properties," *Journal of Geotechnical Engineering (ASCE)*, Vol. 119, No. 1, pp. 147-166. New York: ASCE.

Deutsch, C. V. (2002). Geostatistical Reservoir Modeling. Oxford: Oxford University Press.

DNV (2007). Recommended Practice: Statistical Representation of Soil Data (DNV-RP-C207). Hovik, Norway: Det Norske Veritas.

Dolinar B. (2010). “Predicting the Normalized, Undrained Shear Strength of Saturated Fine-Grained Soils using Plasticity-Value Correlations,” *Applied Clay Science*, Vol. 47, Nos. 3-4, pp.428-432.

Duncan, M. J. (2000). “Factors of Safety and Reliability in Geotechnical Engineering,” *Journal of Geotechnical and Geoenvironmental Engineering (ASCE)*, Vol. 126, No. 4, pp. 307-317. Reston, VA: ASCE.

Ejezie, S. U., and Harrop-Williams, K. (1984). “Principles of Probabilistic Characterizations of Soil Properties”. In *Probabilistic Characterization of Soil Properties: Bridge Between Theory and Practice*, D. S. Bowles, and H-H. Ko (eds.), pp. 140-156. New York: ASCE.

Elkateb, T., Chalaturnyk, R., and Robertson, P. K. (2003a). “An Overview of Soil Heterogeneity: Quantification and Implications on Geotechnical Field Problems,” *Canadian Geotechnical Journal*, 40: 1-15.

Elkateb, T., Chalaturnyk, R., and Robertson, P. K. (2003b). “Simplified Geostatistical Analysis of Earthquake-Induced Ground Response at the Wildlife Site, California, U.S.A,” *Canadian Geotechnical Journal*, 40: 16-35.

El-Ramly, H., Morgenstern, N. R., and Cruden, D. M. (2002). “Probalistic Slope Stability Analysis for Practice,” *Canadian Geotechnical Journal*, 39: 665-683.

Fenneman, N.M. (1938). Physiography of the Eastern United States. New York: McGraw-Hill.

Fenton, G. A. (1999). “Estimation for Stochastic Soil Models,” *Journal of Geotechnical and Geoenvironmental Engineering (ASCE)*, Vol. 125, No. 6, pp. 470-485. Reston, VA: ASCE.

Fenton, G. A., and Griffiths, D. V. (2008). Risk Assessment in Geotechnical Engineering. Hoboken, NJ: Wiley.

Geologismiki (2010). CPeT-IT version 1.6. http://www.geologismiki.gr/ (10-2-2010).

Hahn, G. J, and Shapiro, S. S. (1994). Statistical Models in Engineering, Wiley Classic Library Edition. New York: Wiley.

Haldar, A., and Mahadevan, S. (2000). Probability, Reliability, and Statistical Methods in Engineering Design. New York: Wiley.

Harr, M. E. (1987). Reliability-Based Design in Civil Engineering. New York: McGraw-Hill.

Hight, D. W. (2004). "Sampling Effects in Soft Clay: An Update on Ladd and Lambe (1963)," *Soil Behavior and Soft Ground Construction*, Proceedings of Symposia in Honor of Charles C. "Chuck" Ladd (GSP 119). pp. 86-121. Reston, VA: ASCE.

Isaaks, E. H., and Srivastava, R. M. (1989). An Introduction to Applied Geostatistics. Oxford: Oxford University Press.

Jaksa, M. B., Brooker, P. I. and Kaggwa, W. S. (1997a). "Inaccuracies Associated with Estimating Random Measurement Errors," *Journal of Geotechnical and Geoenvironmental Engineering (ASCE)*, Vol. 123, No. 5, pp. 393-401. New York: ASCE.

Jaksa, M. B., Brooker, P. I. and Kaggwa, W. S. (1997b). "Modeling Spatial Variability of the Undrained Shear Strength of Clay Soils using Geostatistics." In *Geostatistics Wollongong '96*, E. Y. Baafi and N. A. Schofield (eds.), Volume 2, pp. 1284-1295. Amsterdam: Kluwer.

Jaksa, M. B., Kaggwa, W. S., and Brooker, P. I. (1993). "Geostatistical Modeling of the Spatial Variation of the Shear Strength of a Stiff, Overconsolidated Clay." In *Proceedings of the Conference on Probabilistic Methods in Geotechnical Engineering*, K. S. Li and S. C. Lo (eds.), Canberra, Australia, February 10-12, pp.185-194. Rotterdam: Balkema.

Jaksa, M. B., Kaggwa, W. S., and Brooker, P. I. (2000). "Experimental Evaluation of the Scale of Fluctuation of a Stiff Clay." In *Proceedings of the 8th International Conference on Applications of Statistics and Probability to Soil and Structural Engineering*, R. E. Melchers, and M. G. Stewart (eds.), Sydney, December 12-15, 1999, pp. 415-422. Rotterdam: Balkema.

Jamiolkowski, M., Ladd, C. G., Germaine, J. T., and Lancellotta, R. (1985). "New Development in Field and Laboratory Testing of Soils," *Proceedings of the 11th International Conference on Soil Mechanics and Foundation Engineering*, San Francisco, Vol. 1, pp. 57-153.

Jones, A. L., Kramer, S. L., and Arduino, P. (2002). "Estimation of Uncertainty in Geotechnical Properties for Performance-Based Earthquake Engineering." *PEER Report 2002-16*. Berkeley, CA: Pacific Earthquake Engineering Center.

Journel, A. G., and Huijbregts, CH. J. (1978). Mining Geostatistics. London: Academic Press.

Karlsson, R. and Viberg, L. (1967). "Ratio c/p' in Relation to Liquid Limit and Plasticity Index with Special Reference to Swedish Clays." *Proceedings of the Geotechnical Conference*, Oslo, Norway, Vol. 1, pp. 43-47. Oslo: Norwegian Geotechnical Institute.

Keaveny, J. M., Nadim, F., and Lacasse, S. (1989). “Autocorrelation Functions for Offshore Geotechnical Data.” *Structural Safety and Reliability,* Proceedings of ICOSSAR ’89, the 5th International Conference on Structural Safety and Reliability, San Francisco, August 7-11, 1989), A. H. -S. Ang, M. Shinozuka, and G. I. Schuëller, (eds.). pp. 263-270. New York: ASCE.

Kelkar, M., and Perez, G. (2002). Applied Geostatistics for Reservoir Characterization. Richardson, Texas: Society for Petroleum Engineers.

Kulatilake, P. H. S. W., and Gosh, A. (1988). “An investigation into the accuracy of Spatial Variation estimation using static cone penetration data.” In *Proceedings of Penetration Testing 1988,* First International Symposium on Penetration Testing, ISOPT-1, J. De Ruiter (ed.), pp.815-821. Rotterdam: AA Balkema.

Kulatilake, P. H. S. W., and Um, J-G. (2003). “Spatial Variation of Cone Tip Resistance for the Clay Site at Texas A&M University,” *Geotechnical and Geological Engineering*, Volume 21, pp. 149-165. Amsterdam: Kluwer.

Kulhawy, F.H. and Mayne, P.W. (1990), “Manual on Estimating Soil Properties for Foundation Design,” *Report No. EL-6800.* Palo Alto, CA: Electric Power Research Institute.

Lacasse, S. and Nadim, F. (1996). “Uncertainties in Characterizing Soil Properties.” In *Uncertainty in the Geological Environment: From Theory to Practice*, Geotechnical Special Publication No. 58, C. D. Shackleford, P. P. Nelson, and M. J. S. Roth (eds.), pp 49 - 75. New York: ASCE.

Ladd, C. C. and DeGroot, D. J. (2003). “Recommended Practice for Soft Ground Site Characterization.” *The Arthur Casagrande Lecture*, Proceedings of the 12th Panamerican Conference on Soil Mechanics and Geotechnical Engineering, Boston, MA, Vol. 1, pp. 3-57. Essen, Germany: Verlag.

Ladd, C. C., and Lamb, T. W. (1963). “The Strength of Undisturbed Clay Determined from Undrained Tests.” *Symposium on Laboratory Shear Testing of Soils,* ASTM STP 361, pp. 342-371. Philadelphia, PA: American Society for Testing and Materials.

Ladd, C.C. and DeGroot, D.J. (2003). “Recommended Practice for Soft Ground Site Characterization.” *The Arthur Casagrande Lecture*, Proceedings of the 12th Panamerican Conference on Soil Mechanics and Geotechnical Engineering, Boston, MA, Vol. 1, pp. 3-57. Essen, Germany: Verlag.

Loehr, E. J., Finley, C. A., and Huaco, D. (2006). “Procedures for Design of Earth Slopes using LRFD.” *Report No.: OR 06-010*. Jefferson City, Mo: MoDOT.

Low, B. K. (2005). "Reliability-Based Design Applied to Retaining Walls," *Geotechnique*, Vol. 55, No. 1, pp. 63-75. London: ICE.

Lumb, P. (1966). "The Variability of Natural Soils," *Canadian Geotechnical Journal*, Volume 3, No. 2, pp. 74-79.

Lumb, P. (1970). "Safety Factors and the Probability Distribution of Soil Strength," *Canadian Geotechnical Journal*, 7 (3): 225-242.

Lumb, P. (1974). "Application of Statistics in Soil Mechanics." In Soil Mechanics – New Horizons, I. K. Lee (ed.), pp. 44-111. New York: Elsevier.

Lunne, T., Powell, J.J.M., and Robertson, P.K. (1998). Cone Penetration Testing in Geotechnical Practice. London: E&FN Spon

Maerz, N. and Magner, K. (2010). Cone Penetrometer Data Analysis, *Draft Report*, MTI/MoDOT Transportation Geotechnics Research Program.

Matheron, G. (1963). "Principles of Geostatistics," *Economic Geology*, Vol. 58, pp. 1246 – 1266.

McBratney, A. B., and Webster, R. (1986). "Choosing Functions for Semi-Variograms of Soil Properties and Fitting them to Sample Estimates," *European Journal of Soil Science*, 37: 617-639. Oxford: Wiley-Blackwell.

Meek, D. W. (2001). "A Semi-Parametric Method for Estimating the Scale of Fluctuation," *Computer and Geosciences*, 27: 1243-1249.

Mesri, G. (1975). "Discussion: New Procedure for Stability of Soft Clays," *Journal of Geotechnical Engineering, ASCE*, 101(GT4), pp. 409-412. New York: ASCE.

Middendorf, M.A. (2003). Geologic Map of Missouri, 2003: Sesquicentennial Edition, Missouri Department of Natural Resources, Division of Geology and Land Survey, Missouri State Geologic Maps SGM-2003, Rolla, Missouri, Scale 1:500000.

Minasny, B., McBratney, A.B., and Whelan, B.M. (2005). "VESPER version 1.62." Australian Centre for Precision Agriculture, The University of Sydney, NSW 2006. *http://www.usyd.edu.au/su/agric/acpa*. (08/16/2010).

Minoru, M., and Shogaki, T., (1988). "Effects of Plasticity and Sample Disturbance on Statistical Properties of Undrained Shear Strength," *Soils and Foundations*, Vol.28, No.2, pp.14-24. Tokyo: Japanese Society of Soil Mechanics and Foundation Engineering.

MODGLS (2007). Missouri Environmental Geology Atlas. Missouri Department of Natural Resources (MODNR), Division of Geology and Land Survey (DGLS), [CD-ROM], Rolla, Missouri.

Nadim, F. (1988). "Geotechnical Site Description using Stochastic Interpolation." *Norwegian Geotechnical Institute Publication No. 176*, pp. 1-4. Oslo: NGI.

Nadim, F. (2007). "Tools and Strategies for Dealing with Uncertainty in Geotechnics". In *Probabilistic Methods in Geotechnical Engineering*, (*CISM International Centre for Mechanical Sciences, No.491*), D. V. Griffiths and G. A. Fenton (eds.), pp. 71-95. New York: Springer.

Nagaraj, T. S., and Srinivasa Murthy, B. R. (1986). "A Critical Reappraisal of Compression Index," *Geotechnique*, 36(1), pp. 27-32. London: ICE.

NIST (2010). NIST/SEMATECH e-Handbook of Statistical Methods. http://www.itl.nist.gov/div898/handbook/. (02 August 2010).

Nobre, M. M., and Sykes, J. F. (1992). "Application of Bayesian Kriging to Subsurface Characterization," *Canadian Geotechnical Journal*, 29: 589-598.

Pearson, E. S. and Hartley, H. O. (1972). Biometrika Tables for Statisticians Volume 2. Cambridge: University Press.

Pearson, E. S. and Hartley, H. O. (1976). Biometrika Tables for Statisticians Volume 1. London: Biometrika Trust.

Pearson, K. (1956). Early Statistical Papers. Cambridge: University Press.

Phoon, K. K., and Kulhawy, F. H. (1999a). "Characterization of Geotechnical Variability," *Canadian Geotechnical Journal*, 36: 612-624.

Phoon, K. K., and Kulhawy, F. H. (1999b). "Evaluation of Geotechnical Property Variability," *Canadian Geotechnical Journal*, 36: 625-639.

Phoon, K. K., Kulhawy, F.H. and Grigoriu, M. D. (1995). "Reliability-Based Design of Foundations for Transmission Line Structures." *Report No. TR-105000.* Palo Alto, CA: Electric Power Research Institute.

Phoon, K. K., Kulhawy, F.H. and Grigoriu, M. D. (2003). "Development of a Reliability-Based Design of Foundations for Transmission Line Foundations," *Journal of Geotechnical and Geoenvironmental Engineering (ASCE)*, Vol. 129, No. 9, pp. 798-806. Reston, VA: ASCE.

Phoon, K.-K., Quek, S.-T., and An, P. (2004). "Geostatistical Analysis of Cone Penetration Test (CPT) Sounding using the Modified Bartlett Test," *Canadian Geotechnical Journal*, 41: 356-365.

Rackwitz, R. (2000). "Reviewing Probabilistic Soil Modeling," *Computers and Geotechnics*, 26: 199-223. Amsterdam: Elsevier.

Rethati, L. (1988). Probabilistic Solutions in Geotechnics. Amsterdam: Elsevier.

Saucier, R.T. (1994). *Geomorphology and Quaternary Geologic History of the Lower Mississippi Valley*. U.S. Army Corps of Engineers Waterways Experiment Station, Vicksburg, MS.

Saville, V. B., and Davis, W. C. (1962). Geology and Soils Manual. Jefferson City, MO: Missouri State Highway Commission.

Skempton, A. W. (1944). "Notes on the Compressibility of Clays," *Quarterly Journal of the Geological Society of London*, 100 (C: Parts 1 and 2), pp. 119-135. London: Geological Society.

Skempton, A. W., and Henkel, D. J. (1953). "The Post-Glacial Clays of the Thames Estuary at Tilbury and Shellhaven." *Proceedings, 3rd International Conference on Soil Mechanics and Foundation Engineering*, Zurich, Switzerland, Vol. 1, pp. 302-308.

Skempton, A.W. (1957). "Discussion: The Planning and Design of the New Hong Kong Airport." *Proceedings of the Institution of Civil Engineers, London*, Vol. 7, pp. 305–307. London: ICE.

Smith, G. N. (1986). Probability and Statistics in Civil Engineering: An Introduction. New York: Nichols.

Snedecor, G. W., and Cochran, W. G. (1964). Statistical Methods. Iowa City, Iowa: University of Iowa Press.

Soulie, M. (1984). "Geostatistical Applications in Geotechnics." In *Geostatistics for Natural Resources Characterization* (NATO ASI Series C, Volume 122), Part 2, G. Verly, M. David, A. G. Journel, and A. Marechal (eds), pp. 703-730. Dordrecht, Holland: J. Reidel Publishing.

Soulie, M., Montes, P., and Silvestri, V. (1990). "Modeling Spatial Variability of Soil Parameters," *Canadian Geotechnical Journal*, 27: 617-630.

Srivastava, A., and Sivakumar Babu, G.L. (2009). "Effect of Soil Variability on the Bearing Capacity of Clay and in Slope Stability Problems," *Engineering Geology*, 108: 142–152.

Stephenson, R. W. (2009a). “Lecture Notes, CE 414: Measurement of Soil Properties.” Department of Civil, Architectural, and Environmental Engineering, University of Science and Technology, Rolla, Missouri

Stephenson, R. W. (2009b). “Lecture Notes, CE 329: Foundation Engineering II.” Department of Civil, Architectural, and Environmental Engineering, University of Science and Technology, Rolla, Missouri.

Tan, C. P., Donald, I. B., and Melchers, R. E. (1993). “Probalistic Slope Stability Analysis: State-of-Play.” In *Proceedings of the Conference on Probabilistic Methods in Geotechnical Engineering*, K. S. Li and S. C. Lo (eds.), Canberra, Australia, February 10-12, pp. 89-110. Rotterdam: AA Balkema.

Tang, W. H. (1984). “Principles of Probabilistic Characterizations of Soil Properties.” In *Probabilistic Characterization of Soil Properties: Bridge Between Theory and Practice*, D. S. Bowles, and H-H. Ko, (eds.), pp. 74-89. New York: ASCE.

Trauner, L., Dolinar, B., and Misic, M., (2005). “Relationship between the Undrained Shear Strength, Water Content, and Mineralogical Properties of Fine-Grained Soils,” *International Journal of Geomechanics, ASCE,* Vol. 5, pp. 350-355. Reston, VA: ASCE.

Unlu, K., Nielsen, D. R., Biggar, J. W., and Morkoc, F. (1990). “Statistical parameters characterizing the spatial variability of selected hydraulic properties,” *Soil Science Society of America Journal*, 54: 1537 – 1547. Madison,WI: SSSA.

Uzielli, M., Lacasse, S., Nadim, F. and Phoon, K. K. (2007). “Soil Variability Analysis for Geotechnical Practice.” In *Characterization and Engineering Properties of Natural Soils*, T. S. Tan, K. K. Phoon, D. W. Hight, and S. Leroueil (eds.), pp.1653-1752. London: Taylor and Francis.

Uzielli, M., Vannucchi, G., and Phoon, K. K. (2005). “Random Field Characterization of Stress-Normalised Cone Penetration Testing Parameters,” *Geotechnique*, 55: No 1: pp. 3-20. London: ICE.

Vanmarcke, E. H. (1977). “Probabilistic Modeling of Soil Profiles,” *Journal of Geotechnical Engineering (ASCE)*, Vol. 103, No. 11, pp. 1227 – 1246. New York: ASCE.

Vanmarcke, E. H. (1983). Random Fields: Analysis and Synthesis. Cambridge, MA: MIT Press.

VESPER. (2010). Vesper 1.6 User manual www.usyd.edu.au/agriculture/acpa/documents/Vesper_1.6_User_Manual.pdf. (8-16-2010).

Web-Reg. (2011). Correlogram. www.web-reg.de *(10-10-2011).*

White, W. (1993). “Soil Variability: Characterization and Modeling.” In *Proceedings of the Conference on Probabilistic Methods in Geotechnical Engineering*, K. S. Li and S. C. Lo (eds.), Canberra, Australia, February 111-120, pp.233-236. Rotterdam: AA Balkema.

Whitfield, J. W. (1982). Surficial Materials Map of Missouri, Missouri Department of Natural Resources, Division of Geology and Land Survey, Rolla, Missouri.

Whitman, R. V. (1996). “Organizing and Evaluating Uncertainty in Geotechnical Engineering.” In *Uncertainty in the Geological Environment: From Theory to Practice*, Geotechnical Special Publication No. 58, C. D. Shackleford, P. P. Nelson, and M. J. S. Roth (eds.), pp 1 - 28. New York: ASCE.

Wickremesinghe, D. S., and Campanalla, R. G. (1993). “Scale of Fluctuation as a Descriptor af Soil Variability.” In *Proceedings of the Conference on Probabilistic Methods in Geotechnical Engineering*, K. S. Li and S. C. Lo (eds.), Canberra, Australia, February 10-12, pp.233-236. Rotterdam: AA Balkema.

Wood, D. M., and Worth, C. P. (1978). “The Correlation of Index Properties with some Basic Engineering Properties of Soils,” *Canadian Geotechnical Journal*, 15(2): 137-145.

Worth, C. P. and Houlsby, G. T. (1985). “Soil Mechanics-Property Characterization and Analysis Procedures.” *Proceedings of the 11th International Conference on Soil Mechanics and Foundation Engineering*, Vol. 1, San Francisco, pp.1-55.

Yin, J. H. (1999). “Properties and Behaviour of Hong Kong Marine Deposits with Different Clay Contents,” *Canadian Geotechnical Journal*, Vol. 36, pp. 1085-1095.

VITA

Sitenikechukwu Onyejekwe was born in Okigwe, Nigeria. In November 1989, he received his B.Eng. with Honors in Civil Engineering from the University of Nigeria, Nsukka, Nigeria. Upon graduation he worked in civil engineering consulting in Nigeria. In January 2006, he received his M.Phil. in Civil Engineering from the University of Birmingham, UK. In May 2012, he received his Ph.D. in Civil Engineering from the Missouri University of Science and Technology, Rolla, Missouri, USA.